

*DISCRIMINABILITY AND SENSITIVITY TO REINFORCER MAGNITUDE
IN A DETECTION TASK*

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Three pigeons discriminated between two sample stimuli (intensities of red light). The difficulty of the discrimination was varied over four levels. At each level, the relative reinforcer magnitude for the two correct responses was varied across conditions, and the reinforcer rates were equal. Within levels, discriminability between the sample stimuli did not change systematically as reinforcer magnitude varied. Across levels, the sensitivity of behavior to changes in the reinforcer-magnitude ratio decreased as the discriminability between the sample stimuli increased. Subsequent analysis showed that this relation was limited to performance following only one of the sample stimuli, the dim red light that remained constant across all conditions. Extant behavioral models of signal detection cannot easily accommodate these results.

Key words: signal detection, sensitivity, discriminability, reinforcer magnitude, key peck, pigeon

Signal detection and identification are fundamental aspects of behavior. For example, birds must decide if a passing butterfly is edible or toxic, motorists must decide whether it is safe to overtake, and pathologists must decide whether a certain cell is cancerous or not. Behavioral models of detection attempt to describe these decisions in terms of stimulus control (the effects of the psychophysical disparity between the stimuli) and reinforcer control (the effects of the consequences for choices).

Davison and Tustin (1978) provided an extension to the model of two-alternative choice behavior known as the generalized matching law (Baum, 1974) to account for signal-detection performance. The generalized matching law (GML) can be written,

$$\frac{B_1}{B_2} = c \left(\frac{R_1}{R_2} \right)^{a_r}, \quad (1)$$

or, in its logarithmic form, as

$$\log \frac{B_1}{B_2} = a_r \log \frac{R_1}{R_2} + \log c, \quad (2)$$

where B_1 and B_2 are the numbers of responses on two alternatives, and R_1 and R_2 are the

corresponding numbers of reinforcers obtained. The parameter a_r measures the sensitivity of behavior to changes in the reinforcer ratio (Lobb & Davison, 1975). The parameter c , inherent bias, measures any constant preference for an alternative across changes in the reinforcer ratio (Baum, 1974). For example, this might arise due to undetected asymmetries in the apparatus (e.g. the two operanda might require slightly different forces) or in the organism (e.g., handedness in human subjects).

Davison and Tustin (1978) suggested that a detection task was an extended form of a two-alternative choice task. After each presentation of a sample stimulus (S_1 or S_2), there are two possible responses (B_1 and B_2) that provide occasional reinforcement following a correct response. When S_1 and S_2 are indiscriminable, the procedure is a form of a concurrent schedule, and Equation 1 should describe performance. As the stimuli become more discriminable, however, the subjects should make more B_1 responses following S_1 presentations, and more B_2 responses following S_2 presentations. Davison and Tustin proposed two equations to describe this effect. Following S_1 presentations, this took the form

$$\log \frac{B_{11}}{B_{12}} = a_r \log \frac{R_{11}}{R_{22}} + \log c + \log d, \quad (3)$$

and following S_2 presentations,

$$\log \frac{B_{21}}{B_{22}} = a_r \log \frac{R_{11}}{R_{22}} + \log c - \log d, \quad (4)$$

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where B_{11} and B_{12} are the numbers of responses on Alternatives 1 and 2 following S_1 presentations, and B_{21} and B_{22} are the corresponding numbers of responses following S_2 presentations. The reinforcer terms, R_{11} and R_{22} , are the numbers of reinforcers obtained for correct responses, B_{11} and B_{22} , and the parameters a_r and c are the same as in Equations 1 and 2. The parameter $\log d$, stimulus discriminability, measures the preference for correct responses, B_{11} in Equation 3 and B_{22} in Equation 4.

Equations 3 and 4 assume independencies between reinforcer control and stimulus control in detection tasks. Algebraic subtraction of Equations 3 and 4 gives an equation for discriminability,

$$\frac{1}{2} \log \frac{B_{11}B_{22}}{B_{12}B_{21}} = \log d, \quad (5)$$

where all notation is as above. Equation 5 predicts that discriminability between the sample stimuli is independent of the reinforcer ratio and inherent bias. This prediction usually has been upheld (Davison & McCarthy, 1988), although a reanalysis of past studies (Johnstone & Alsop, 1999) found some evidence that more extreme reinforcer distributions might lead to higher estimates of discriminability than equal reinforcer distributions.

Algebraic summation of Equations 3 and 4 gives an equation for response bias ($\log b$),

$$\frac{1}{2} \log \frac{B_{11}B_{21}}{B_{12}B_{22}} = a_r \log \frac{R_{11}}{R_{22}} + \log c, \quad (6)$$

where all notation is as above. Equation 6 predicts that the sensitivity of behavior to the ratio of obtained reinforcers (a_r) and inherent bias ($\log c$) are independent of stimulus discriminability ($\log d$). The status of this prediction is more uncertain. Figure 1 plots the relation between sensitivity to the reinforcer ratio and stimulus discriminability from some past studies. These graphs show only the results when $\log d$ was less than 1.75 because very high discriminabilities produce error rates so low that it is difficult to get accurate parameter estimates.

The earliest study, McCarthy and Davison (1980a), found no relation between these parameters. A larger study by McCarthy and Davison (1984) produced conflicting results.

In one half of their study, they used a controlled reinforcer ratio procedure that ensured the obtained reinforcer ratio was nearly identical to the arranged reinforcer ratio (e.g., Stubbs & Pliskoff, 1969). When the reinforcer ratio was varied across different levels of stimulus disparity, there was no systematic variation between a_r and $\log d$. The other half of their study arranged uncontrolled reinforcer ratios that allowed the obtained reinforcer ratio to deviate from the arranged reinforcer ratio depending on subjects' performance. This procedure produced a systematic decrease in a_r as $\log d$ increased. The difference between these results cannot be attributed solely to the use of controlled and uncontrolled procedures because Alsop and Davison (1991) later found an inverse relation between a_r and $\log d$ using a controlled procedure. Godfrey (1997) also found an inverse relation between a_r and $\log d$ across two levels of stimulus disparity when the stimuli signaling the choice alternatives were keys backlit by 5% or 95% densities of white pixels against a black background. The same study, however, found no such relation across four levels of sample stimulus disparity when the choice alternatives were signaled by 35% and 65% densities of pixels, and the absolute level of a_r was sufficient such that this result could not be attributed simply to some sort of floor effect (see also Godfrey & Davison, 1998, for a different treatment of some of these data).

All of the studies in Figure 1 varied reinforcer frequency in order to arrange different consequences for the two classes of correct response. In fact, nonhuman animal research in this area has focused almost exclusively on the effects of relative reinforcer frequency on response bias. Relative reinforcer duration or magnitude has received far less attention, even though it often is cited as an effective biaser in human psychophysical studies (e.g., Dusoir, 1983; Green & Swets, 1974). Hume (1974) found that rats were biased toward the alternative that gave the greater number of brain stimulations for each correct response, but the size of this effect was not large. Boldero, McCarthy, and Davison (1985) compared the effectiveness of reinforcer frequency and reinforcer duration as biasers of detection performance. They varied reinforcer duration over three levels (3 s vs. 3 s, 5 s vs. 1.5 s, and 1.5 s vs. 5 s) and reinforcer frequency over five

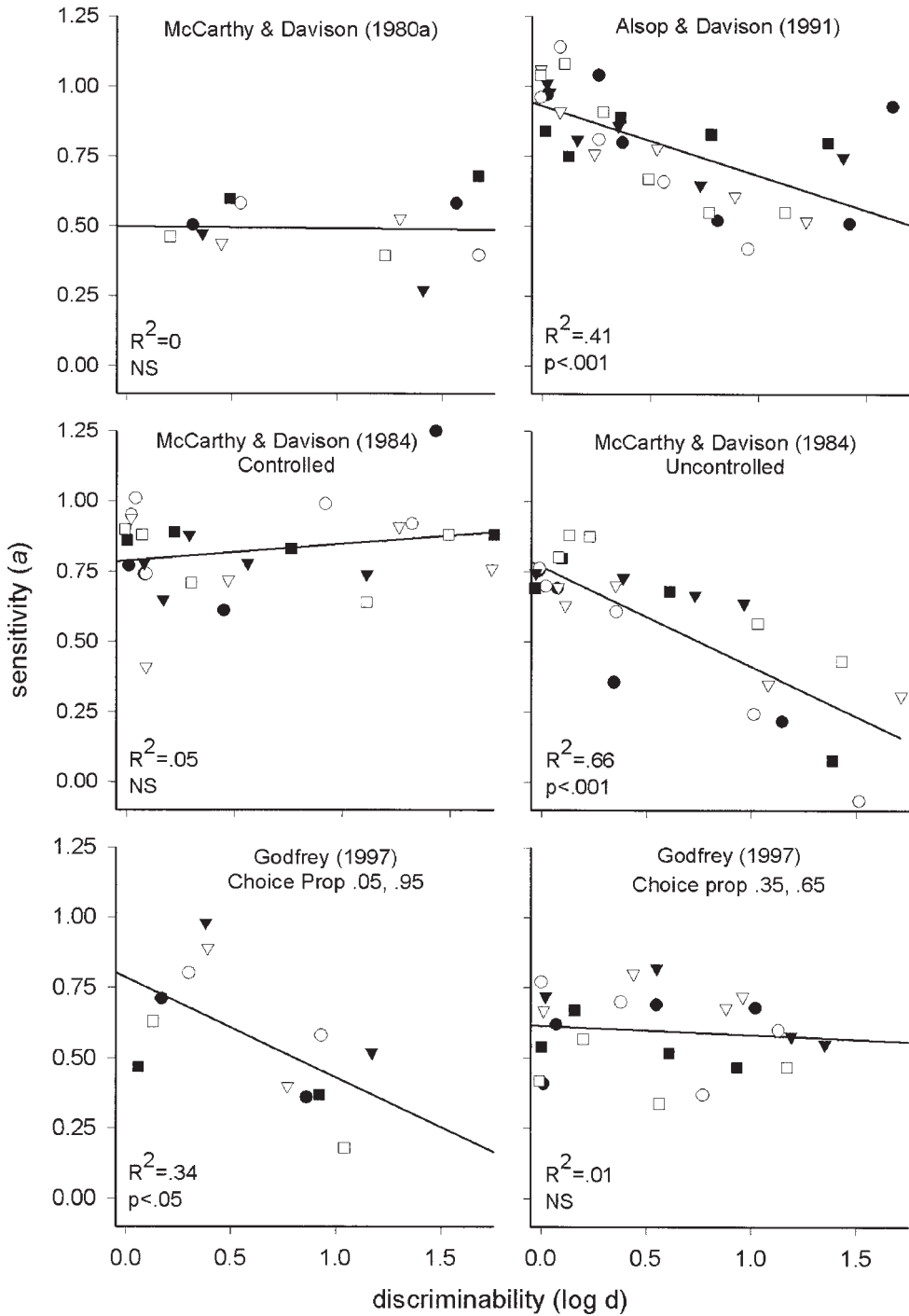


Fig. 1. Sensitivity of response bias to changes in the reinforcer ratio (a , Equation 6) plotted as a function of discriminability between the sample stimuli ($\log d$, Equation 5). The separate panels show the results from a number of past experiments, and the different symbols signify different subjects (see text for details).

levels across their 15 experimental conditions. They analysed their results using simple extensions to Equations 3, 4, and 6 based on Baum and Rachlin's (1969) concatenation of the GML. Equation 3 became

$$\log \frac{B_{11}}{B_{12}} = a_r \log \frac{R_{11}}{R_{22}} + a_m \log \frac{M_{11}}{M_{22}} + \log c + \log d, \quad (7)$$

Equation 4 became

$$\log \frac{B_{21}}{B_{22}} = a_r \log \frac{R_{11}}{R_{22}} + a_m \log \frac{M_{11}}{M_{22}} + \log c - \log d, \quad (8)$$

and Equation 6 became

$$\frac{1}{2} \log \frac{B_{11}B_{21}}{B_{12}B_{22}} = a_r \log \frac{R_{11}}{R_{22}} + a_m \log \frac{M_{11}}{M_{22}} + \log c, \quad (9)$$

where M_{11} and M_{22} refer to the magnitude of reinforcers for correct responses, the parameter a_m measures the sensitivity of behavior to the ratio of those magnitudes, and all other notation is as above.

Boldero et al. (1985) used Equation 9 in a multiple-regression analysis to assess the contribution of reinforcer frequency and reinforcer magnitude on response bias. Reinforcer frequency had a significant effect on behavior, and the mean sensitivity of behavior to the ratio of reinforcer frequencies (a_r in Equation 9) was 0.63. In contrast, although the sensitivity of behavior to the ratio of reinforcer magnitudes was positive for all subjects (mean $a_m = 0.24$), the estimate of a_m for each pigeon did not differ significantly from zero by standard statistical criteria. Boldero et al. presumed that a wider variation of reinforcer durations would have led to values of a_m that were significantly different from zero for individual subjects. They also reported that the estimates of discriminability ($\log d$, Equation 5) were independent of changes in the relative reinforcer durations. This final result warrants some caution, however, because the analysis of response bias suggested that reinforcer magnitude had not been an effective independent variable in this study.

The scant research concerning the effects of reinforcer magnitude on detection limits the generality of behavioral models of detection,

especially regarding its application to corresponding human research. For example, the interaction between sensitivity to the reinforcer ratio and discriminability found in some research (e.g., Figure 1) might be peculiar to manipulations of relative reinforcer frequency. Likewise, detection studies that show estimates of discriminability do not change when reinforcer characteristics vary (i.e., Equation 5) typically have manipulated reinforcer frequency. In fact, research using a related task (delayed matching-to-sample) has found higher discriminability when reinforcer magnitudes are unequal than when they are equal (e.g., Jones, White, & Alsop, 1995).

The present study examined the relation between relative reinforcer magnitude, response bias, and discriminability in a detection task. It arranged four sets of conditions. Within each set, the relative duration of the reinforcers (M_{11} and M_{22}) was varied across three levels. Across sets of conditions, the physical difference between the sample stimuli was varied to produce four levels of stimulus discriminability. The results addressed three questions. First, were changes in relative reinforcer duration sufficient to produce changes in response bias? Second, and more important, was the sensitivity of behavior to changes in the relative reinforcer duration independent of stimulus discriminability? Finally, were estimates of discriminability within each set of conditions independent of changes in relative reinforcer duration?

METHOD

Subjects

Three ex-racing pigeons, numbered 41, 42, and 43, were maintained at 85% to 90% of their free-feeding body weights. Most of their food was obtained during the experimental procedure, but mixed grain was provided if necessary. Water and grit were freely available. All pigeons had prior experience on conditional discrimination tasks. Pigeon 42 died toward the end of Condition 14, and it was not replaced.

Apparatus

The pigeons' home cages doubled as experimental chambers with interface panels forming the back wall. The cages had dimensions of

500 mm by 500 mm by 500 mm. The interface panels had three circular, translucent keys (25 mm diameter) set in a horizontal row, 70 mm apart from each other and 240 mm from the cage floor. A red (635 nm) light-emitting diode (LED) light bar (RS Electronics #109-387) was behind the center key. For Stimulus S_1 , two of the eight LEDs were lit, and for Stimulus S_2 , two (i.e., $S_1 = S_2$), three, four, or six of the LEDs were variously lit, depending on the particular condition. There was also a white LED behind the center key, and a green LED behind each of the side keys. A 50 mm high by 60 mm wide opening in the interface situated 100 mm below the center key provided access to a food magazine containing wheat. Duration of access to the wheat was 2 s, 4 s, or 6 s, depending on the experimental condition.

The pigeons were in neighboring cages, and there were partitions between adjacent cages extending approximately three quarters of the length of the cage so that a pigeon could not view another pigeon while working on the task. Water, grit, and a perch were located at the front of the cage. The pigeons were housed in a room with one large translucent window, and a floor-length curtain was drawn while the pigeons were working in order to maintain a low level of background light. A 386sx computer running MED-PC[®] software controlled the experimental conditions and recorded data.

Procedure

Each trial began with the center key illuminated white. Immediately following a single peck to the center key, the white light was replaced by a red light of one of two intensities, and the side keys were illuminated green. A single peck to either side key extinguished all the key lights. If the center key was the dimmer red light (S_1), then a peck to the left key was "correct" and was reinforced intermittently. If the center key was the brighter red light (S_2), then a peck to the right key was correct and was reinforced intermittently. In all conditions, the frequency of reinforcement was arranged by a dependent concurrent random-interval (RI) 60-s RI 60-s schedule of reinforcement. More specifically, after each reinforcer a RI 30-s schedule was started, and once a reinforcer was arranged it was assigned to the next correct response

Table 1
Sequence of experimental conditions.

| Stimuli | Condition | Reinforcer duration | | |
|---------|-----------|---------------------|-------|----------------|
| | | Left | Right | Number of days |
| Set A | 1 | 6 s | 2 s | 24 |
| | 2 | 2 s | 6 s | 28 |
| | 3 | 4 s | 4 s | 42 |
| | 4 | 6 s | 2 s | 22 |
| Set B | 5 | 2 s | 6 s | 27 |
| | 6 | 6 s | 2 s | 21 |
| | 7 | 4 s | 4 s | 21 |
| | 8 | 2 s | 6 s | 45 |
| Set C | 9 | 6 s | 2 s | 50 |
| | 10 | 2 s | 6 s | 36 |
| | 11 | 4 s | 4 s | 27 |
| | 12 | 6 s | 2 s | 23 |
| Set D | 13 | 2 s | 6 s | 22 |
| | 14 | 6 s | 2 s | 31 |
| | 15 | 4 s | 4 s | 21 |
| | 16 | 2 s | 6 s | 22 |

following either an S_1 presentation or an S_2 presentation with a probability of $p = .5$. No other reinforcers were arranged until that reinforcer had been obtained, after which the RI schedule timed the next interval. Nonreinforced correct responses and incorrect responses led to 8 s of blackout, and then the next trial commenced. Reinforced correct responses produced X-s access to food followed by Y s of blackout before the next trial commenced, where X and Y always summed to 8 s. The values of X and Y varied across conditions (Table 1). The presentation of S_1 or S_2 on any trial was determined randomly with a probability of $p = .5$.

There were four sets of conditions (Table 1). Within each set, the physical disparity between S_1 and S_2 remained constant, and the reinforcer magnitudes were varied across conditions. The four sets of conditions followed the same general pattern. The first condition arranged unequal reinforcer magnitudes (e.g., 2 s vs. 6 s), the second condition arranged a reversal of these conditions (e.g., 6 s vs. 2 s), the third condition arranged equal reinforcer magnitudes (e.g., 4 s vs. 4 s), and the final condition was a replication of the first condition (Table 1). The stimulus disparity between S_1 and S_2 was varied across sets of conditions. Set D had the most disparate stimuli, followed by Set A, Set C, and Set B (in which S_1 and S_2 were identical).

Experimental sessions were conducted twice daily (7:00 a.m. and 4:00 p.m.). Each session ended after 250 trials were completed or when 90 min elapsed, whichever came first. The pigeons were weighed and fed, if necessary, at approximately 9:30 a.m. each day. Conditions were changed after 21 days (i.e., 42 sessions) unless there were conspicuous trends in the last 7 days' estimates of $\log d$ and $\log b$. If trends were apparent, training continued until the data appeared stable.

For each session, the data collected were the number of responses to the left and right keys following S_1 presentations (B_{11} and B_{12}) and S_2 presentations (B_{21} and B_{22}) and the number of food reinforcers obtained for correct responses on the left (R_{11}) and right (R_{22}) keys.

RESULTS

Figure 2 plots the logarithm (\log) of the ratio of responses following S_1 presentations ($\log B_{11}/B_{12}$) and S_2 presentations ($\log B_{21}/B_{22}$) as a function of the logarithm of the ratio of reinforcer magnitudes ($\log M_{11}/M_{22}$). The performance of each pigeon at each level of stimulus disparity is shown in a separate panel. Regression lines were fitted to these data using Equations 8 and 9 with the term for reinforcer frequency dropped from these equations because the arranged reinforcer rates on the alternatives were equal (i.e., $\log R_{11}/R_{22} \approx 0$). Although individual performance varied, choice typically followed the same pattern. When the sample stimuli were indiscriminable (Set B), performances following S_1 and S_2 were virtually identical, and all pigeons' choice showed clear control by the changes in reinforcer magnitude. As the stimulus disparity increased across sets of conditions (Sets C, A, and D), $\log B_{11}/B_{12}$ and $\log B_{21}/B_{22}$ diverged, reflecting increased discriminability between the stimuli. At the higher levels of stimulus disparity (Sets A and D), the effect of reinforcer-magnitude ratio was smaller and more variable following S_1 presentations (filled symbols) than following S_2 presentations (open symbols). This result will be revisited below.

Figure 3 plots estimates of stimulus discriminability ($\log d$, Equation 5) for each pigeon in each condition as a function of the ratio of reinforcer magnitudes ($\log M_{11}/M_{22}$). The

estimates of discriminability also are given in Table 2. There are two aspects of the data to consider. First, as stimulus disparity increased across sets of conditions, estimates of $\log d$ typically increased, but there were individual differences in the relation between physical and psychophysical disparity. For example, estimates of $\log d$ were similar for Sets A and C for Pigeon 41, for Sets B and C for Pigeon 42, and for Sets A and D for Pigeon 43. Second, there was no evidence of any systematic relation between the ratio of reinforcer magnitudes and estimates of stimulus discriminability (Figure 2, Table 2). The only consistent change in discriminability within sets of conditions seemed to be an order effect; that is, estimates of $\log d$ for the first conditions of Sets A and C were consistently lower than those for the other conditions in those sets (Table 2).

Figure 4 plots estimates of response bias ($\log b$, Equation 6) for each pigeon in each condition as a function of the ratio of reinforcer magnitudes ($\log M_{11}/M_{22}$). Separate linear regressions were conducted for each set of conditions. Although there was some individual variability, Figure 4 shows a reasonably clear pattern. At lower stimulus disparities, changes in reinforcer magnitude produced orderly changes in $\log b$, and the sensitivity of behavior to changes in the ratio of reinforcer magnitudes (a_m) was similar to those obtained in similar concurrent-schedule procedures (e.g., Davison & McCarthy, 1988); the mean estimates of a_m across pigeons were 0.68, 0.59, and 0.52 for Sets B, C, and A, respectively. At the highest stimulus disparity (Set D), the data were generally less orderly, and the estimates of a_m were lower (mean = 0.35).

Figure 5 summarizes the relation between stimulus disparity, stimulus discriminability ($\log d$), and sensitivity of behavior to changes in the ratio of reinforcer magnitude (a_m). The top left panel shows a monotonic increase in $\log d$ (averaged across conditions within each set) as stimulus disparity increased. The top middle panel generally shows a decrease in a_m as stimulus disparity increased. The values of a_m from Set B (lowest stimulus disparity) were, with one exception (Pigeon 42, Set C), greater than those from Sets C, A, and D. Similarly, the values of a_m from Sets C and B (intermediate stimulus disparity) were always greater than those from Set D. Some of the irregularities in

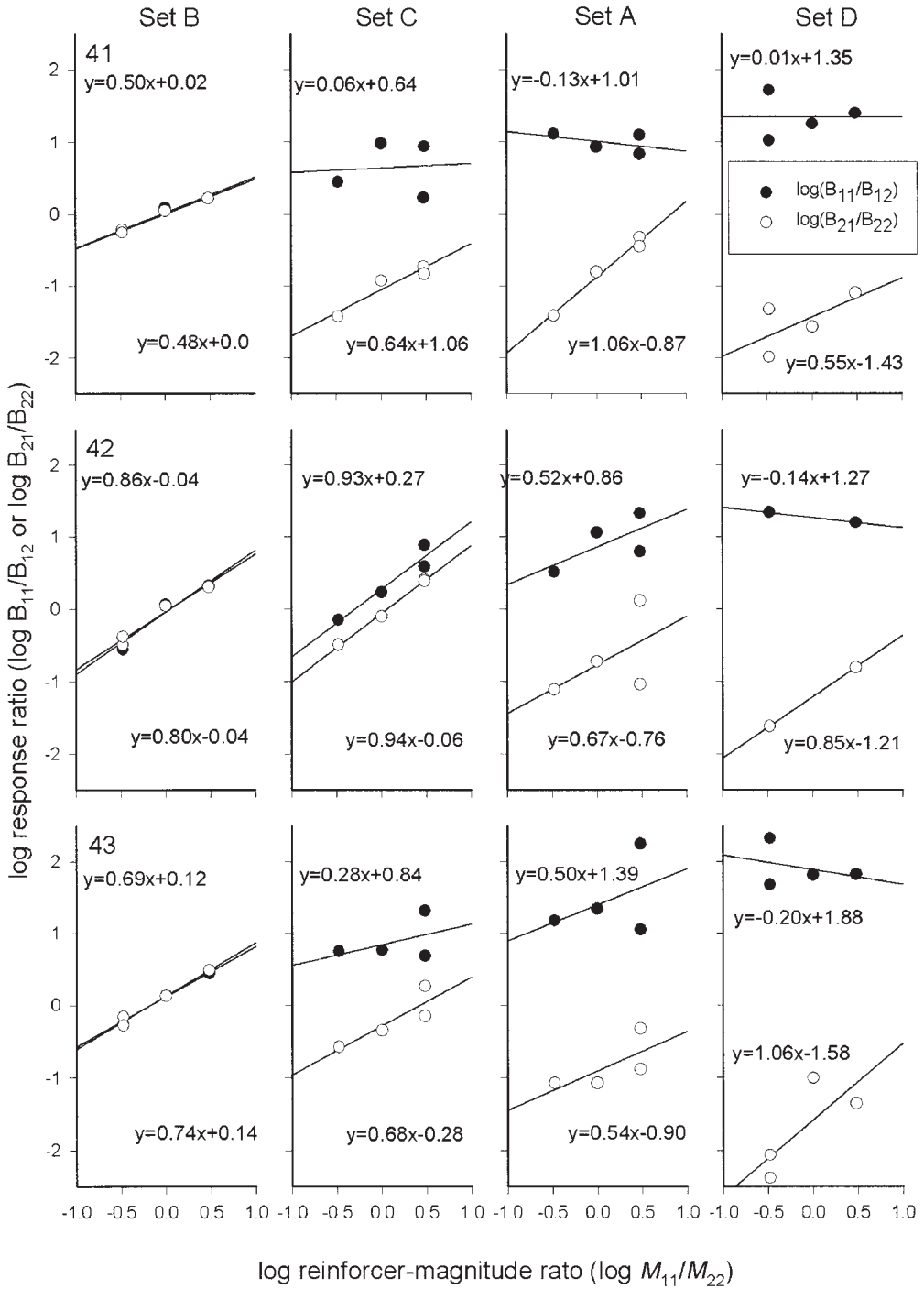


Fig. 2. The log response ratios following S_1 presentations (log B_{11}/B_{12}) and following S_2 presentations (log B_{21}/B_{22}) plotted as a function of the log reinforcer-magnitude ratio (log M_{11}/M_{22}) for each pigeon in each set of experimental conditions.

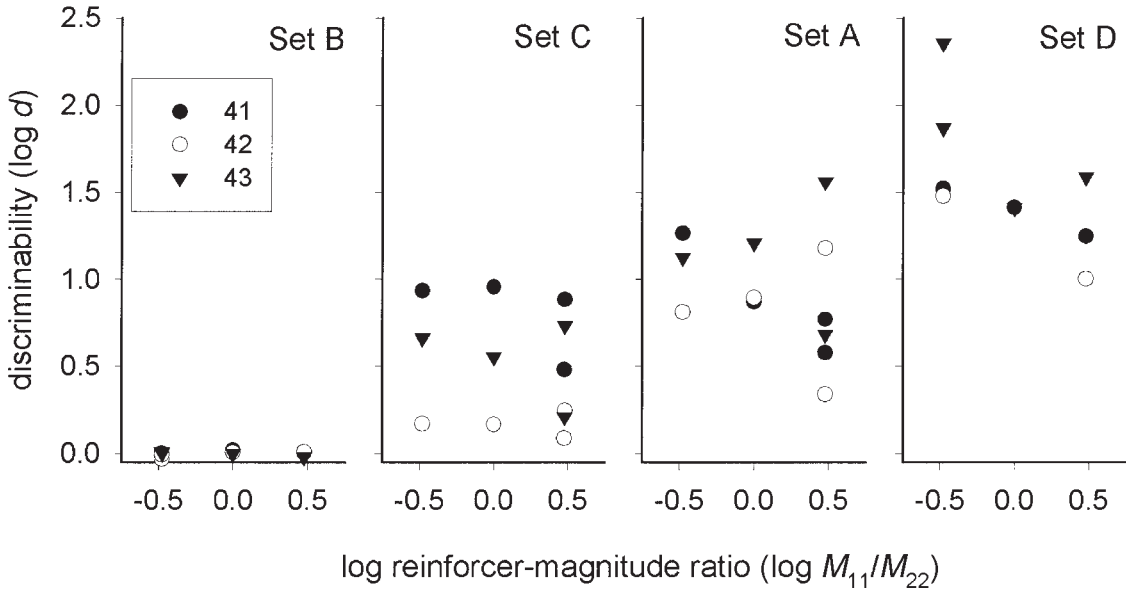


Fig. 3. Estimates of discriminability between the sample stimuli ($\log d$, Equation 5) plotted as a function of the log ratio of reinforcer magnitudes ($\log M_{11}/M_{22}$) for each pigeon in each set of experimental conditions.

the estimates of a_m across sets of conditions seemed related to individual differences in discrimination between the various sample stimuli. For Set C, for example, Pigeon 42's results showed not only an unusually high estimate of a_m (top middle panel), but also a low discriminability between the sample stimuli (top left panel). For this reason, the bottom left panel of Figure 5 plots a_m as a function of $\log d$, and it also allows a direct comparison of these data with those in Figure 1. There is a clear, and statistically significant ($R^2 = .50$, $p < .025$), inverse correlation between these two measures; this result suggests that stimulus discriminability and the sensitivity of behavior to the ratio of reinforcer magnitudes were not independent.

One aspect of the data in Figure 2 prompted further analysis. It appeared that the decrease in sensitivity to reinforcement as stimulus disparity increased was more pronounced following S_1 presentations than S_2 presentations. To test this possibility, Figure 5 (bottom middle and right panels) plots sensitivity to reinforcement as a function of stimulus discriminability (i.e., the same analysis as in the bottom left panel of Figure 5) separately for S_1 and S_2 performance. Figure 5 shows that the interaction between a_m and $\log d$ was only significant for performance following S_1 .

DISCUSSION

There were three main findings. First, changes in reinforcer magnitude were effective in producing changes in response bias during the detection task (Figures 2 and 4), and this effect was larger and more reliable than that obtained by Boldero et al. (1985). Figure 5 provides a possible explanation for the clearer effects found in the present study. Boldero et al. arranged a relatively high level of stimulus discriminability in all their conditions ($\log d \approx 1.5$). Figure 5 suggests that Boldero et al. were unlikely to find large effects of changes in reinforcer magnitude at these levels of stimulus discriminability due to the inverse relation between sensitivity to reinforcer magnitude and stimulus discriminability.

Second, the present study found no evidence that estimates of discriminability ($\log d$) covaried with changes in reinforcer magnitude (Figure 3); that is, the independence between stimulus control and reinforcer control predicted by Equation 5 was supported. On the one hand, this result is encouraging because it is consistent with the Davison and Tustin (1978) model. On the other hand, it might be unexpected to readers who are familiar with the differential outcome effect (DOE) sometimes reported in delayed matching-to-sample (DMTS) procedures. A major

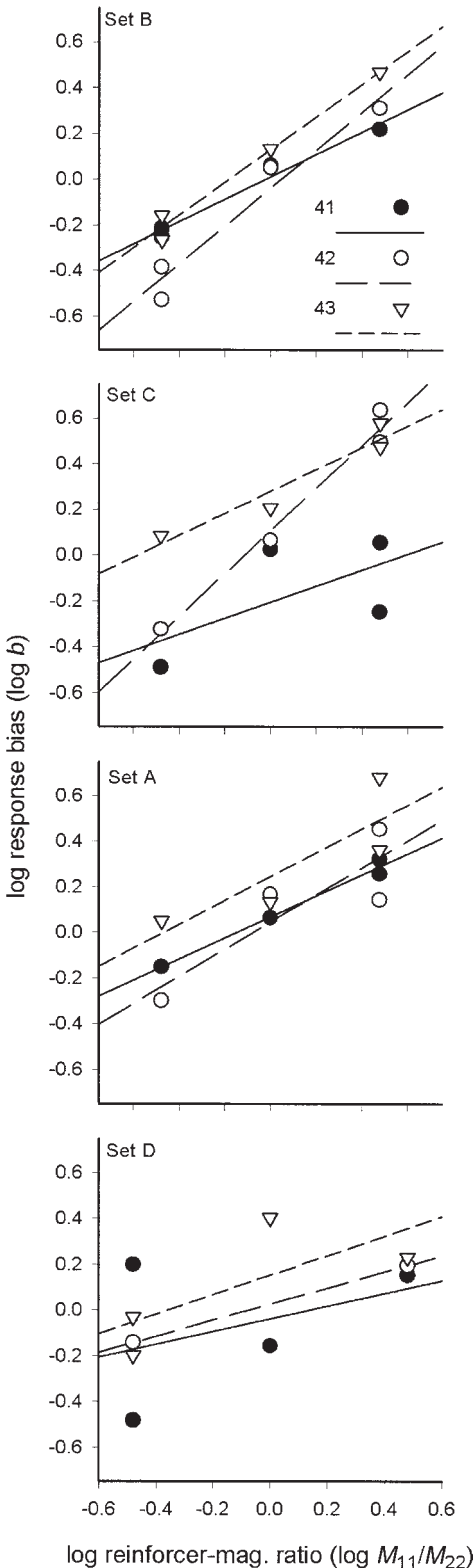
Table 2

The number of responses to each response alternative (B_{11} , B_{12} , B_{21} , and B_{22}) and the number of reinforcers (R_{11} and R_{22}) obtained for correct responses following each stimulus in each condition. Estimates of discriminability ($\log d$) and response bias ($\log b$) for each condition also are shown, as is the sensitivity of reinforcement (a) to changes in reinforcer magnitude within each set of conditions.

| Pigeon | Condition | B_{11} | B_{12} | B_{21} | B_{22} | R_{11} | R_{22} | $\log d$ | $\log b$ | (a) |
|--------|-----------|----------|----------|----------|----------|----------|----------|----------|----------|---------|
| 41 | 1 | 914 | 134 | 338 | 708 | 220 | 244 | 0.58 | 0.26 | 0.46 |
| | 2 | 849 | 65 | 33 | 864 | 233 | 231 | 1.27 | -0.15 | |
| | 3 | 1,023 | 119 | 156 | 997 | 274 | 272 | 0.87 | 0.06 | |
| | 4 | 1,048 | 84 | 315 | 894 | 275 | 265 | 0.77 | 0.32 | |
| | 5 | 476 | 782 | 470 | 772 | 222 | 200 | 0.00 | -0.22 | |
| | 6 | 749 | 448 | 780 | 473 | 216 | 198 | 0.00 | 0.22 | |
| | 7 | 688 | 571 | 651 | 590 | 187 | 213 | 0.02 | 0.06 | |
| | 8 | 451 | 809 | 442 | 798 | 205 | 201 | 0.00 | -0.26 | |
| | 9 | 718 | 421 | 180 | 965 | 255 | 265 | 0.48 | -0.25 | |
| | 10 | 888 | 319 | 44 | 1,175 | 304 | 311 | 0.94 | -0.49 | |
| | 11 | 963 | 101 | 111 | 944 | 265 | 281 | 0.95 | 0.02 | |
| | 12 | 983 | 113 | 135 | 912 | 241 | 262 | 0.88 | 0.05 | |
| | 13 | 1,207 | 23 | 58 | 1,212 | 329 | 276 | 1.52 | 0.20 | |
| | 14 | 1,062 | 42 | 88 | 1,094 | 286 | 294 | 1.25 | 0.15 | |
| | 15 | 1,032 | 57 | 27 | 1,004 | 272 | 235 | 1.41 | -0.16 | |
| | 16 | 1,063 | 100 | 12 | 1,175 | 306 | 311 | 1.51 | -0.48 | |
| 42 | 1 | 1,079 | 174 | 705 | 542 | 224 | 212 | 0.34 | 0.45 | |
| | 2 | 977 | 300 | 88 | 1,135 | 275 | 263 | 0.81 | -0.30 | |
| | 3 | 1,099 | 96 | 206 | 1,099 | 273 | 263 | 0.89 | 0.17 | |
| | 4 | 1,219 | 58 | 103 | 1,120 | 274 | 272 | 1.18 | 0.14 | |
| | 5 | 264 | 955 | 309 | 972 | 143 | 157 | -0.03 | -0.53 | |
| | 6 | 853 | 409 | 827 | 411 | 202 | 161 | 0.01 | 0.31 | |
| | 7 | 668 | 584 | 656 | 592 | 177 | 201 | 0.01 | 0.05 | |
| | 8 | 357 | 879 | 372 | 892 | 175 | 168 | -0.01 | -0.39 | |
| | 9 | 993 | 259 | 898 | 350 | 182 | 163 | 0.09 | 0.50 | |
| | 10 | 515 | 733 | 304 | 948 | 203 | 185 | 0.17 | -0.32 | |
| | 11 | 821 | 481 | 530 | 668 | 207 | 220 | 0.17 | 0.07 | |
| | 12 | 1,105 | 145 | 888 | 362 | 196 | 180 | 0.25 | 0.64 | |
| | 13 | 1,124 | 52 | 31 | 1,293 | 257 | 297 | 1.48 | -0.14 | |
| | 14 | 1,195 | 76 | 166 | 1,063 | 280 | 256 | 1.00 | 0.20 | |
| 43 | 1 | 1,145 | 103 | 404 | 848 | 260 | 269 | 0.68 | 0.36 | |
| | 2 | 1,156 | 77 | 98 | 1,169 | 264 | 251 | 1.13 | 0.05 | |
| | 3 | 1,184 | 54 | 93 | 1,109 | 273 | 297 | 1.21 | 0.13 | |
| | 4 | 528 | 3 | 61 | 467 | 144 | 147 | 1.56 | 0.68 | |
| | 5 | 235 | 337 | 234 | 339 | 96 | 87 | 0.00 | -0.16 | |
| | 6 | 355 | 126 | 361 | 117 | 85 | 66 | -0.02 | 0.47 | |
| | 7 | 398 | 294 | 385 | 282 | 119 | 116 | 0.00 | 0.13 | |
| | 8 | 92 | 166 | 90 | 170 | 39 | 47 | 0.01 | -0.27 | |
| | 9 | 1,017 | 210 | 804 | 437 | 226 | 198 | 0.21 | 0.47 | |
| | 10 | 980 | 176 | 253 | 956 | 273 | 278 | 0.66 | 0.08 | |
| | 11 | 749 | 130 | 265 | 591 | 210 | 185 | 0.55 | 0.21 | |
| | 12 | 822 | 40 | 364 | 518 | 196 | 180 | 0.73 | 0.58 | |
| | 13 | 1,262 | 6 | 5 | 1,211 | 297 | 279 | 2.35 | 0.03 | |
| | 14 | 527 | 8 | 23 | 521 | 139 | 149 | 1.59 | 0.23 | |
| | 15 | 1,042 | 16 | 95 | 965 | 236 | 248 | 1.41 | 0.40 | |
| | 16 | 1,170 | 25 | 10 | 1,170 | 296 | 290 | 1.87 | -0.20 | |

difference between a DMTS procedure and the current study is that the sample stimulus is removed and there is a delay period (sometimes called the retention interval) before the subject makes a choice response in a DMTS procedure. The DOE occurs if accuracy is greater when the outcomes for the two choice responses are

dissimilar (different qualities or magnitudes of reinforcement) than when the outcomes are identical. The absence of a DOE in the present study probably is not surprising because it is not a true DMTS procedure; there was no retention interval, and the sample stimulus was present while the pigeons made their choice responses.



Third, the present study found that sensitivity to changes in reinforcer magnitude ratio decreased as discriminability between the stimuli increased (Figure 5). This result conflicts with Davison and Tustin's (1978) model (Equations 6 and 9) that predicts independence between these measures. It is, however, consistent with the results from those previous studies that found an inverse relation between stimulus discriminability and sensitivity to reinforcer-rate ratio (Figure 1). This suggests that these previous results are not peculiar to reinforcer-rate ratio, but reflect a more general relation between stimulus discriminability and reinforcement in detection procedures.

Alsop and Davison's (1991) behavioral model of signal detection has had some success accounting for the effects of stimulus discriminability and response bias without interactions between model parameters (see Davison & Nevin, 1999). This model was based on Davison and Jenkins's (1985) model of concurrent-schedule performance, which is given by

$$\frac{B_1}{B_2} = c \left(\frac{R_1 + \frac{R_2}{d_{br}}}{\frac{R_1}{d_{br}} + R_2} \right), \quad (10)$$

where B , R , c , and the subscripts 1 and 2 are as above, and the parameter d_{br} (ranging from 1.0 to infinity) measures the subject's discriminability or confusion between the two response-reinforcer contingencies. As discriminability between the response-reinforcer contingencies decreases, the extent that any one reinforcer generalizes across both alternatives increases. Alsop and Davison's extension of this model for signal detection treats the effects of stimulus discriminability (d_{sb} in their model) in a similar manner to d_{br} . Following S_1 , this model can be written

$$\frac{B_{11}}{B_{12}} = c \left(\frac{R_{11} + \frac{R_{22}}{d_{sb}d_{br}}}{\frac{R_{11}}{d_{br}} + \frac{R_{22}}{d_{sb}}} \right), \quad (11)$$

←

Fig. 4. Estimates of response bias ($\log b$, Equation 6) plotted as a function of the log ratio of reinforcer magnitudes ($\log M_{11}/M_{22}$) for each pigeon in each set of experimental conditions.

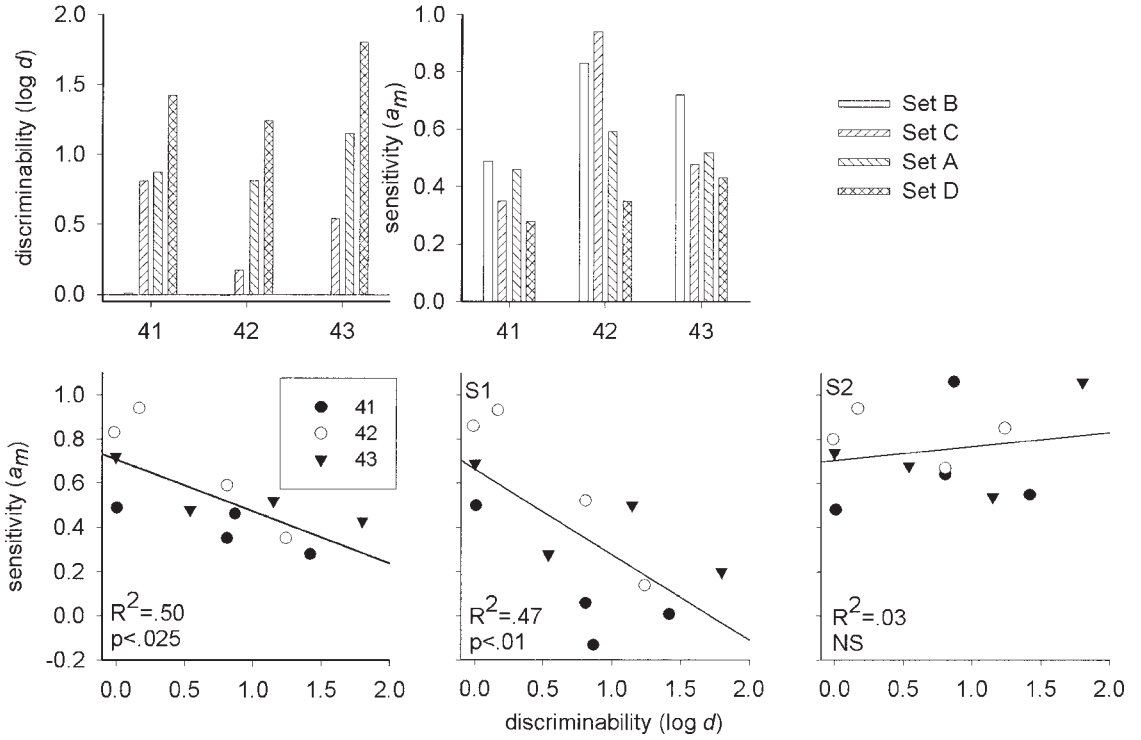


Fig. 5. The average discriminability ($\log d$, Equation 5) of each pigeon within each set of conditions (top left panel) and the sensitivity of response bias to changes in the ratio of reinforcer magnitudes (a_m , Equation 6; top middle panel) plotted for each set of conditions. The bottom left panel plots these estimates of sensitivity against the corresponding estimates of discriminability for each pigeon in each set of conditions. Sensitivity following S_1 ($\log B_{11}/B_{12}$; bottom middle panel) and following S_2 presentations ($\log B_{21}/B_{22}$; bottom right panel) to changes in the ratio of reinforcer magnitudes for each set of conditions also are plotted separately as a function of the corresponding estimates of discriminability between the stimuli.

and following S_2 by

$$\frac{B_{21}}{B_{22}} = c \left(\frac{\frac{R_{11}}{d_{sb}} + \frac{R_{22}}{d_{br}}}{R_{22} + \frac{R_{11}}{d_{sb}d_{br}}} \right), \quad (12)$$

where all notation is as above. Davison and Nevin described d_{sb} and d_{br} as measuring “the generalization engendered by confusion between stimulus-response relations and between response-reinforcer relations” (p. 449) in a signal-detection procedure, and they presented a reanalysis of a number of past studies demonstrating and extending the versatility of the Alsup and Davison model.

Whereas the GML concatenates to include the effects of relative reinforcer magnitude readily (e.g., Equations 7 and 8), it is less clear how to extend Equations 10, 11, and 12 to accommodate factors other than reinforcer

frequency. Davison and Nevin (1999) presented two possible extensions of Davison and Jenkins’s (1985) concurrent-schedule model that incorporate reinforcer magnitude, but they noted shortcomings with both. Of the two extensions, the equation that they appeared to favor on empirical grounds took the form,

$$\frac{B_1}{B_2} = c \left(\frac{R_1 + \frac{R_2}{d_{br}}}{\frac{R_1}{d_{br}} + R_2} \right) \left(\frac{M_1 + \frac{M_2}{d_{bm}}}{\frac{M_1}{d_{bm}} + M_2} \right), \quad (13)$$

where d_{bm} measured the discriminability of the relation between responses and the values of the reinforcer magnitudes M_1 and M_2 , and all other notation is as above. They also added the proviso that d_{bm} should be a function of the ratio or difference between M_1 and M_2 , for reasons that will become apparent below.

Davison and Nevin did not present equations to describe signal-detection performance, but Equation 13 leads logically to the following model:

$$\frac{B_{11}}{B_{12}} = c \left(\frac{R_{11} + \frac{R_{22}}{d_{sb}d_{br}}}{\frac{R_{11}}{d_{br}} + \frac{R_{22}}{d_{sb}}} \right) \left(\frac{M_{11} + \frac{M_{22}}{d_{sb}d_{bm}}}{\frac{M_{11}}{d_{bm}} + \frac{M_{22}}{d_{sb}}} \right), \quad (14)$$

and

$$\frac{B_{21}}{B_{22}} = c \left(\frac{\frac{R_{11}}{d_{sb}} + \frac{R_{22}}{d_{br}}}{R_{22} + \frac{R_{11}}{d_{sb}d_{br}}} \right) \left(\frac{\frac{M_{11}}{d_{sb}} + \frac{M_{22}}{d_{bm}}}{M_{22} + \frac{M_{11}}{d_{sb}d_{bm}}} \right), \quad (15)$$

where all notation is as above. There are obstacles to applying Equations 14 and 15 to the results of the present study, however. In the GML analysis (Equations 7 and 8), the term describing the contribution of reinforcer frequency (i.e., $a_r \log[R_{11}/R_{22}]$) reduces to zero when R_{11} equals R_{22} , and so the present study can ignore it. This is not the case for Equations 14 and 15. When R_{11} equals R_{22} , Equation 14 becomes

$$\frac{B_{11}}{B_{12}} = c \left(\frac{d_{sb}d_{br} + 1}{d_{sb} + d_{br}} \right) \left(\frac{M_{11} + \frac{M_{22}}{d_{sb}d_{bm}}}{\frac{M_{11}}{d_{bm}} + \frac{M_{22}}{d_{sb}}} \right), \quad (16)$$

and Equation 15 takes a similar form. In other words, an analysis of the current data using Equations 14 and 15 would require estimating d_{br} even though there were no variations in reinforcer frequency. It is difficult to have confidence in parameter estimates derived from such an analysis; hence it is not performed here. It is worth noting that the same problem presents itself when M_{11} equals M_{22} and reinforcer frequency is varied, as is the case for most previous research in this area (e.g., Alsop & Davison, 1991; McCarthy & Davison, 1979, 1980a). Under these circumstances, Equation 14 becomes

$$\frac{B_{11}}{B_{12}} = c \left(\frac{R_{11} + \frac{R_{22}}{d_{sb}d_{br}}}{\frac{R_{11}}{d_{br}} + \frac{R_{22}}{d_{sb}}} \right) \left(\frac{d_{sb}d_{bm} + 1}{d_{sb} + d_{bm}} \right), \quad (17)$$

and Equation 15 takes a similar form. Equation 17 requires that any detection analysis

measure the effect of discriminability between reinforcer magnitudes, even if the magnitudes are equal (similar difficulties arise for other independent variables such as qualitatively different reinforcers). Davison and Nevin's (1999) proviso that d_{bm} should be a function of the ratio of reinforcer magnitudes (or their difference) seems designed to address this problem. If d_{bm} equals 1 when M_{11} equals M_{22} , then the effects of d_{bm} disappear and Equation 17 reduces to Equation 11.

Another disconcerting aspect of Equations 13, 14, and 15 concerns the extent to which d_{br} and d_{bm} are independent parameters. For example, the usual method of varying discriminability of the relation between responses and the frequency of reinforcers (i.e., d_{br}) has been to alter the physical disparity between the two concurrently available choice alternatives (Alsop & Davison, 1991; Godfrey & Davison, 1998; Miller, Saunders, & Bourland, 1980). It seems logical that this manipulation also should make it more difficult to discriminate the relation between reinforcer magnitude and the respective response alternatives, but Equation 13 makes the unlikely prediction that even if d_{br} was 1.0 (i.e., the response alternatives were indistinguishable), then differences in reinforcer magnitude would still control behavior. Similarly, a concurrent-schedule procedure that introduced unsignalled delays to reinforcement also should affect both d_{br} and d_{bm} , because both the frequency and magnitude of reinforcers earned on one alternative, but delivered following responses on the other, should influence performance. In fact, it is difficult to conceive of concurrent-schedule procedures that could vary d_{br} without varying d_{bm} . It seems likely, therefore, that if Equation 13, and by extension Equations 14 and 15, are correct, then d_{bm} is a function of d_{br} , or d_{bm} and d_{br} at least share a common factor. Whichever, what seems clear is that the complexity of this class of models must increase markedly to incorporate the effects of variables other than reinforcer frequency.

Davison and Jenkins's (1985) and Alsop and Davison's (1991) models are not the only possible direction for theoretical development in this area. For instance, the GML (Equation 1) could be modified to accommodate effects such as changes in the discriminability between the response alternatives or unsignalled reinforcer delays. Such an equation could take

the form,

$$\frac{B_1}{B_2} = c \left[\left(\frac{R_1}{R_2} \right)^{a_r} \left(\frac{M_1}{M_2} \right)^{a_m} \right]^{a_{br}} \quad (18)$$

where all notation is as above, and the parameter a_{br} measures sensitivity to response-reinforcer relations. When a_{br} equals zero, both reinforcer frequency and magnitude exert no differential control. When a_{br} equals 1.0, the differential effects of different reinforcer frequencies or magnitudes are scaled in the way usually described by the GML (Baum, 1974). Equation 18 can be expanded to account for the effects of other independent variables. Some variables (e.g., different reinforcer qualities) would be sensitive to response-reinforcer relations (a_{br}), whereas other factors would not (e.g., different response forces or topographies).

Equation 18 can be extended to describe signal-detection performance. Stimulus discriminability, d , must be a function of the physical disparity between S_1 and S_2 and the sensitivity of behavior to that disparity, a_s . Davison and Tustin (1978) proposed that this stimulus function might take a ratio form, $(S_1/S_2)^{a_s}$, whereas McCarthy and Davison (1980b) found that a modified difference relation, $[(S_1 - S_2)/S_2]^{a_s}$, provided a better fit to their data. Although both of these particular formulations suffer practical limitations, it remains true that a stimulus function should exist for a given stimulus dimension. Furthermore, the sensitivity of choice to that stimulus function also must depend on sensitivity to the response-reinforcer relations. If, for example, the organism cannot discriminate between the two choice alternatives, then changes in sample stimulus disparity can have no control of behavior. Hence, an equation describing performance following S_1 presentations might be given by

$$\frac{B_{11}}{B_{12}} = c \left[\left(\frac{R_{11}}{R_{22}} \right)^{a_r} \left(\frac{M_{11}}{M_{22}} \right)^{a_m} d \right]^{a_{br}}, \quad (19)$$

and following S_2 presentations by

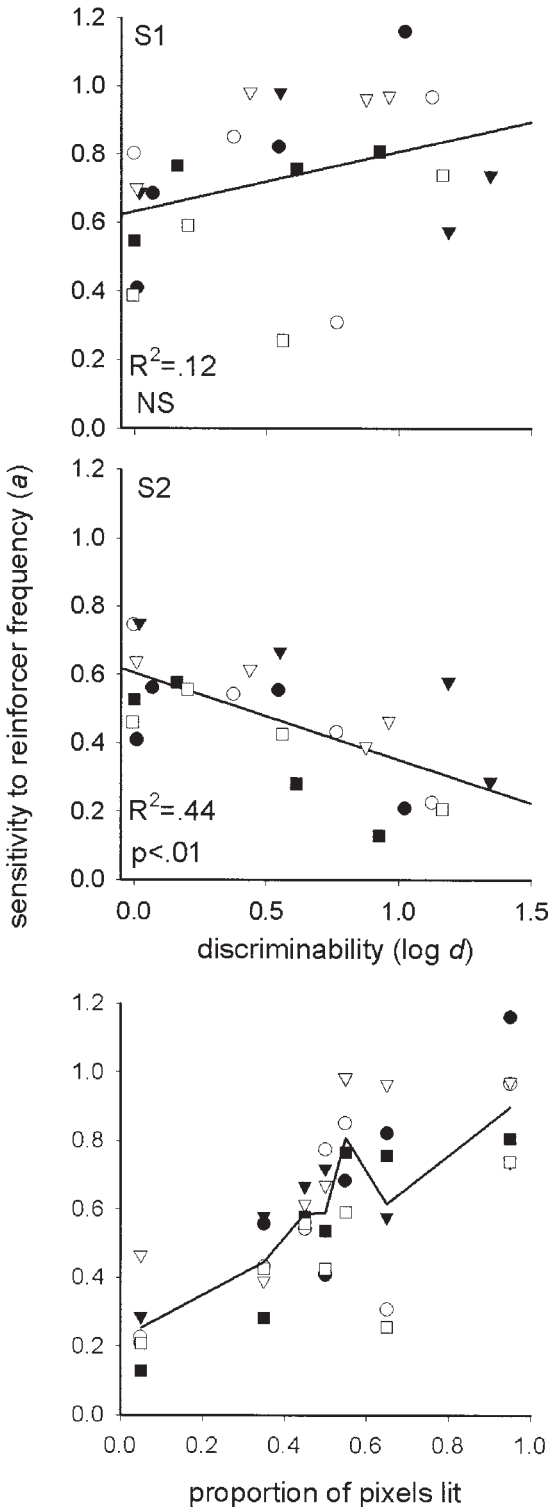
$$\frac{B_{21}}{B_{22}} = c \left[\left(\frac{R_{11}}{R_{22}} \right)^{a_r} \left(\frac{M_{11}}{M_{22}} \right)^{a_m} \frac{1}{d} \right]^{a_{br}}, \quad (20)$$

where all notation is as above. This analysis requires a proviso to accommodate data that show a decrease in the sensitivity of choice to

changes in reinforcer-frequency ratio or magnitude ratio (Figures 1 and 5); that is, the discriminability between S_1 and S_2 affects the sensitivity of choice to response-reinforcer relations, a_{br} . There is a plausible rationale for this proviso. The ratios of reinforcer frequency and magnitude in Equations 19 and 20 refer to reinforcers following both S_1 (R_{11} and M_{11}) and S_2 (R_{22} and M_{22}). As stimulus discriminability increases, sensitivity to response-reinforcer relations that extend across both S_1 and S_2 presentations might be attenuated because sensitivity to local contingencies, signaled by S_1 and S_2 , increases.

Equations 18, 19, and 20 have both strengths and weaknesses compared to Equations 13, 14, and 15. Equation 18 is a reasonably simple extension of a well-established model of choice (i.e., the generalized matching law). It readily accommodates independent variables other than reinforcer frequency, and it can integrate with other recent developments in choice research (e.g., Grace, 1994; Mazur, 2001). Furthermore, Equations 19 and 20 provide a more parsimonious treatment than Equations 14 and 15 when equal frequencies, magnitudes, or qualities of reinforcement are arranged on both choice alternatives; the respective terms simply drop out of the equations. However, Equations 14 and 15 provide a simpler treatment of the interaction between stimulus discriminability and sensitivity to reinforcer frequency, and they probably extend more readily to signal-detection procedures that arrange many stimuli or many response alternatives (see Davison & Nevin, 1999) than Equations 19 and 20.

The final feature of the present results that requires attention concerns the asymmetry in performance following S_1 and S_2 presentations (Figure 5); that is, the sensitivity of behavior to changes in reinforcer magnitude decreased as a function of stimulus discriminability for performance following S_1 stimulus presentations only. McCarthy and Davison (1982) report a similar asymmetry when reinforcer frequency was the independent variable. Using stimulus duration as the dimension to be discriminated, they found that sensitivity to changes in relative reinforcer frequency was greater following presentations of their short stimulus duration than following their long stimulus duration. More recent studies, especially those using the Alsop and Davison



(1991) model, usually do not examine their data for such asymmetries because they tend to fit simultaneously the data following S_1 and S_2 presentations to obtain parameter estimates (e.g., Godfrey & Davison, 1998). Godfrey (1997) provides data that show the same effect as the present experiment, however. Reinforcer frequency was varied at each of four levels of stimulus disparity. The top two graphs in Figure 6 show the sensitivity of behavior to changes in reinforcer frequency plotted as a function of discriminability separately for S_1 (Equation 3) and S_2 (Equation 4) performance. There was a slight, but not statistically significant, increase in estimates of a as $\log d$ increased (top panel) following S_1 presentations. Conversely, there was a statistically significant decrease in a as $\log d$ increased following S_2 presentations. Notice that when S_1 and S_2 performance were analyzed simultaneously (Equation 6), this asymmetry was obscured (Figure 1, bottom left panel).

The reason for these asymmetries (Figures 5 and 6) remains unclear. In more traditional analyses of signal detection (e.g., Green & Swets, 1974), unequal sensitivities to reinforcement following each stimulus would translate as nonunit sloped ROC functions when plotted on normalized coordinates (Davison & McCarthy, 1988; McCarthy & Davison, 1981, 1982). Nonunit ROC slopes have been interpreted as reflecting the transducer function relating physical and sensory continua (e.g., Green & Swets, 1974) or the effects of nonsensory factors such as criterion variance (e.g., Wickelgren, 1968). The present study cannot address such issues, but we do make one observation. In Godfrey's (1997) procedure, the sample stimuli were presented on a small screen behind the center key. The proportion of this screen that was filled with a random pattern of white pixels was varied to produce different stimuli. For the data shown

←

Fig. 6. Sensitivity of the response ratio following S_1 presentations ($\log B_{11}/B_{12}$; top panel) and following S_2 presentations ($\log B_{21}/B_{22}$; middle panel) to changes in the ratio of reinforcer frequency plotted as a function of the corresponding estimates of discriminability between the stimuli for data from Godfrey (1997). The bottom panel plots sensitivity as a function of the proportion of pixels lit for the S_1 or S_2 stimuli. The different symbols denote different subjects.

in the top two panels of Figure 6, the proportion of pixels for S_1 and S_2 stimuli across the four levels of discriminability were .95 versus .05, .65 versus .35, .55 versus .45, and .50 versus .50. The bottom panel of Figure 6 replots the sensitivity a as a function of the proportion of pixels in the sample stimuli. In the case of the .50 versus .50 condition, the means of a following S_1 and S_2 were plotted.

Sensitivity a usually increased as the pixel densities increased. This finding suggests that trial-to-trial variance in the sample stimuli might contribute to Godfrey's results. For example, the perceived intensities of the .05 sample stimulus might vary over a greater range than the other samples because the randomly generated patterns of white pixels could cluster very differently on each trial (one could propose that the proportion of dark pixels was the important factor but, given that dark keys were never reinforced, this seems less likely). A similar argument could be made for McCarthy and Davison's (1980b) finding that estimates of a following a short duration sample stimulus were greater than those following a long duration sample stimulus; that is, the variability in the perception of the longer 3.15-s duration might be greater than the shorter 2.23-s duration. The same rationale could not apply to the present study, however. The changes in a occurred following the stimulus that was held constant throughout the study (S_1), not the stimulus that varied across different disparities (S_2). Although characteristics of the sample stimuli seem an obvious source of these asymmetries, no clear explanation presents itself. It is evident, however, that separate analyses of performance following S_1 and S_2 should be routine.

There are clearly some unresolved issues concerning contemporary behavioral models of signal detection. Although some studies, such as the present experiment, show interactions between a and $\log d$, other studies do not (Figure 1). There seems no clear explanation of these differences between studies, but the asymmetries in performance following different sample stimuli might offer one direction for future research. Furthermore, the present experiment highlights the limitations of contemporary models dealing with independent variables other than reinforcer frequency and stimulus disparity. This is important not only on theoretical grounds, but it also is necessary

before these models can be useful in a wider range of experimental and applied settings where variables such as reinforcer magnitude and quality can be significant factors.

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