

*PREFERENCE BETWEEN VARIABLE-RATIO
AND FIXED-RATIO SCHEDULES:
LOCAL AND EXTENDED RELATIONS*

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Although it has repeatedly been demonstrated that pigeons, as well as other species, will often choose a variable schedule of reinforcement over an equivalent (or even richer) fixed schedule, the exact nature of that controlling relation has yet to be fully assessed. In this study pigeons were given repeated choices between concurrently available fixed-ratio and variable-ratio schedules. The fixed-ratio requirement (30 responses) was constant throughout the experiment, whereas the distribution of individual ratios making up the variable-ratio schedule changed across phases: The smallest and largest of these components were varied gradually, with the mean variable-ratio requirement constant at 60 responses. The birds' choices of the variable-ratio schedule tracked the size of the smallest variable-ratio component. A minimum variable-ratio component at or near 1 produced strong preference for the variable-ratio schedule, whereas increases in the minimum variable-ratio component resulted in reduced preference for the variable-ratio schedule. The birds' behavior was qualitatively consistent with Mazur's (1984) hyperbolic model of delayed reinforcement and could be described as approximate maximizing with respect to reinforcement value.

Key words: choice, concurrent schedules, delay of reinforcement, schedule variability, ratio schedules, key peck, pigeons

Choice between fixed and variable outcomes or food sources has engendered considerable interest across a number of disciplines, ranging from behavioral ecology to the study of reinforcement processes. Many operant studies have assessed such preference with concurrent-chain procedures (e.g., Autor, 1969; Herrnstein, 1964b), in which responding on either of two concurrently available schedules (the initial links of a chain) occasionally results in a stimulus change and corresponding access to one of two outcomes of interest, or terminal links (usually, schedules of food delivery). Preference for one terminal-link schedule over the other may be defined either as the relative rate or time allocation of responding while the initial

links of the chain operate or as the relative rate of entry into each of the terminal links (see Mazur, 1991).

In concurrent-chain studies, preference for variable schedules over fixed schedules of identical average requirement has been a pervasive finding. This preference for variable schedules has been observed with time (Cicerone, 1976), interval (Davison, 1969, 1972; Herrnstein, 1964a; Hursh & Fantino, 1973; Killeen, 1968), and ratio schedules (Duncan & Fantino, 1972; Fantino, 1967) as terminal links, and with rats (Rider, 1983) as well as with pigeons (pigeons have been used more frequently in experiments of this kind).

In studies involving time schedules, preference is assessed between variable and fixed delays from the choice response until the delivery of the terminal-link reinforcer. In interval-schedule studies, preference is assessed between variable- and fixed-interval (FI) terminal links.

Ratio-schedule experiments, in which food in the terminal links occurs following the completion of a given number of responses, have compared fixed-ratio (FR) to mixed- or variable-ratio (VR) schedules of reinforcement. (A mixed schedule can be described as a variable schedule with a small number of components, typically two.) In studies by

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Ahearn (1992) and Ahearn, Hinline, and David (1992), for example, pigeons were given repeated choices between an FR schedule and a VR schedule made up of two or three ratio components. Experimental treatments involved varying the size of the FR schedule or the smallest component in the two- or three-component VR distribution, using minimum requirements of either 1 or 15 responses. The experiments showed that (a) when the smallest component of a VR schedule is set equal to 1, pigeons often choose the VR over the FR schedule, even when the average VR requirement is twice the FR requirement; and (b) this preference reverses if the smallest requirement of the VR schedule is set equal to 15 instead of 1. Assessing the nature of those shifts in preference is the focus of the current study.

In principle, different aspects of a variable schedule could contribute to its being preferred over a fixed one. Cicerone (1976) proposed that variability per se in delays to reinforcement determined preference, a greater range of delays being preferred over a lesser or zero range. But whether variability should be conceived as fundamental is debatable. Although some quantitative models of behavior express a role for variability per se, measured in terms of a conventional summarizing statistic (e.g., Cantor & Wilson, 1981; Grace, 1993), an alternative approach is to focus on the individual components of the variable schedule and on how their respective contributions combine at a behavioral level (e.g., Shull & Spear, 1987). As Herrnstein (1964a) pointed out, in the case of interval or time schedules the adequate combination rule should weight shorter delays more heavily than longer ones. Mazur (1984) has proposed the following rule:

$$V = \sum_{i=1}^n p_i A \left(\frac{1}{1 + kt_i} \right). \quad (1)$$

Assuming that each delay, t_i , appears in the schedule with a probability p_i , Equation 1 identifies the overall value V of the schedule with a weighted average of the individual values of its components. Each component value is composed of the effectiveness of food (A), weighted by the relevant probability and by a decreasing, hyperbolic function of the corresponding delay, t_i . The parameter k deter-

mines the steepness of the delay of reinforcement gradient. Mazur (1986) has proposed a similar treatment for the reinforcing effectiveness of ratio schedules (see also Grossbard & Mazur, 1986). For ratio schedules, the delays, t_i , are the times incurred in schedule completions during the experiment. Mazur's combination rule (Equation 1) implies that the smallest interreinforcement intervals, or ratio components, affect preference in a disproportionate fashion, consistent with earlier suggestions by Davison (1969), Fantino (1967), Sherman and Thomas (1968), and Duncan and Fantino (1970). (See also Schlinger, Blakely, & Kaczor, 1990, for evidence of parallel effects of the smallest ratio components upon postreinforcement pauses.)

Although Mazur's (1984, 1987) studies provided impressive support for Equation 1, they did not rely on the usual concurrent-chain schedules but rather involved an adjusting procedure that offered repeated choices between a variable-component and a single-component alternative. The delay provided by the single-component alternative was increased or decreased until indifference was reached. The reinforcing effectiveness of the variable-component alternative was inferred from the numerical value of the adjusting single-component delay at indifference (see Mazur, 1984, 1987, for more details).

By examining the impact that changes in the smallest and largest VR components have upon choice, the present experiment explores the relative contributions of those component values to the VR schedule's reinforcing effectiveness. It extends the Ahearn (1992) and Ahearn *et al.* (1992) findings to a choice procedure that includes an FR 30 schedule and a four-component VR 60 schedule whose component distribution varies in a parametric fashion. At a more theoretical level, our results are examined by applying a version of Mazur's (1984, 1987) model to choice proportions obtained under concurrent chains of the type used in our study (cf. Davison, 1988; Davison & Smith, 1986; Shull, 1992).

METHOD

Subjects. The subjects were 6 White Carneau pigeons (designated C3, C4, C7, C10, C11, and F3). Each bird's prior experimental his-

tory involved various concurrent FR and VR reinforcement schedules, and no additional shaping was necessary. One additional bird became ill after participating for 2 months and was subsequently removed from the experiment. When not in the experimental chambers, the birds were individually housed in stainless steel cages with free access to water and grit. They were maintained at 80% of their free-feeding weights by means of measured feeding at the end of each working day and during weekends.

Apparatus. The experiment was conducted in four identical standard operant conditioning chambers (Loveland/Gerbrands) for pigeons. Each chamber was equipped with two keys, each mounted 22 cm above the floor on the back wall, with a food hopper centered at the base of the same wall. When operative, the two side keys were transilluminated either yellow or red by 2-W 28-Vdc bulbs with translucent plastic lens covers. Reinforcement consisted of 2.75-s access to mixed grain. Hopper access was accompanied by the illumination of a 2-W bulb inside the hopper unit. Experimental procedures were arranged by means of a Walter-Palya digital controller (Walter & Palya, 1984) that was interfaced with an IBM®-compatible computer (Pevey, 1988). The programming software was a version of real-time BASIC (Walter & Palya, 1985).

Procedure. Each session involved discrete trials and permitted repeated choices between an FR 30 and a four-component VR 60 schedule. Based on prior work by Ahearn (1992) and Ahearn et al. (1992), these requirements were selected so that preference would be likely to vary widely as a function of experimental manipulations. On each trial, the two side keys were illuminated yellow and red, with the color positions randomized. The yellow and red colors accompanied the FR 30 and VR 60 schedules, respectively. Each session began with eight single-chain trials (often referred to as forced-choice trials) that exposed the subject to that session's alternative schedules. On each of these trials, the two side keylights flashed for 0.5 s; then one keylight went dark while the other remained on. Completion of the relevant schedule (the FR 30 or the schedule component of the VR 60) ended in food delivery and initiated the next trial. The very first trial involved the FR or the VR schedule with a .5 probability. The

FR and VR schedules then strictly alternated, each component of the VR schedule being sampled with a .25 probability and without replacement. Thus, once the eight single-chain trials were completed, the bird had been exposed to the FR 30 four times and had also been exposed to each of the four components that composed the current VR 60 schedule.

Following the eight single-chain trials, each session involved 40 concurrent-chain choice trials. During the concurrently available initial links (always FI 3-s schedules), the two side keylights were illuminated. After 3 s had elapsed, the next peck on the yellow or red key produced access to the corresponding terminal link (FR 30 or VR 60 schedule), with the other key going dark. The selected key remained lit, and the corresponding schedule (for the FR 30) or schedule component (for the VR 60) remained operative until food was produced. Food delivery was followed immediately by onset of the next trial. In each successive block of four VR choices, the components of the VR schedule were sampled with a probability $p_i = .25$ and without replacement. That is, irrespective of intervening choices of the FR schedule, each block of four VR choices included all of the four VR components in a random order.

Across the different phases of the experiment, each bird was exposed twice to 15 versions of the four-component VR 60 schedule. As portrayed in Figure 1, the two middle components of the VR schedule remained constant at 30 and 90 responses, respectively. However, the size of the smallest VR component varied in increments of 1, between 1 and 15, while the size of the largest component varied in an inverse fashion between 120 and 106, such that the average VR requirement remained constant at 60.25 responses throughout the whole experiment. We designate the VR schedule simply as a VR 60, for convenience. Also, we identify a given VR schedule distribution by the size of its smallest component. Hence Distributions 1-30-90-120, 2-3-90-119, 3-30-90-118, . . . , 15-30-90-106 will be called Distributions 1, 2, 3, . . . , 15, respectively. Half of the birds (C3, C4, and F3) were exposed to the VR schedule distributions in the ascending and then descending order (1 to 15 followed by 15 to 1; see Figure 1, top). The other half (C7, C10, and

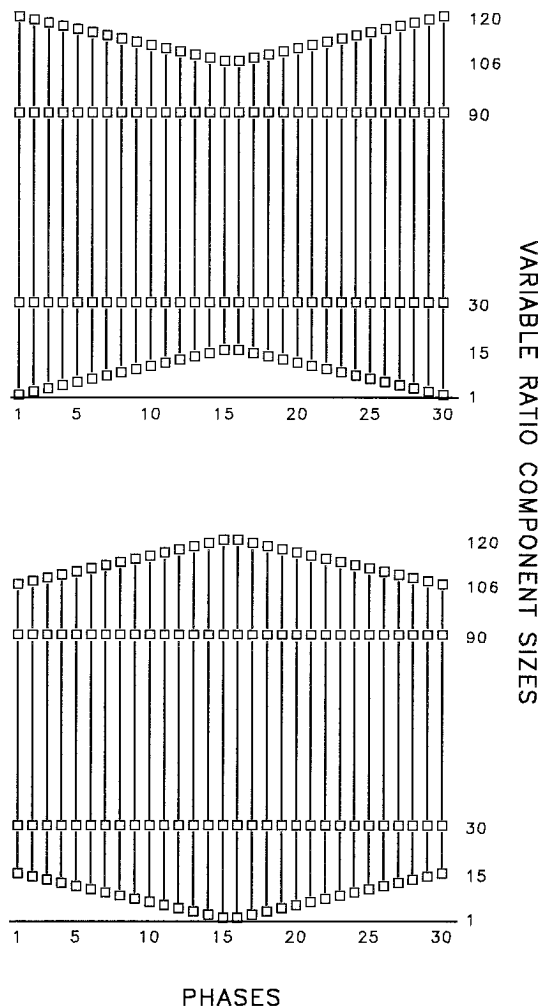


Fig. 1. Distributions of components in the VR 60 schedule for each experimental phase and for each group of birds. The distributions for the ascending-descending group (C3, C4, F3) are shown in the top graph and those for the descending-ascending group (C7, C10, C11) are shown in the bottom graph.

C11) were exposed to the VR distributions in the reverse order, descending then ascending (15 to 1 followed by 1 to 15, see Figure 1, bottom). Each VR distribution was in effect for 11 consecutive sessions within a given experimental phase.

In summary, each bird was exposed twice to 15 different VR distributions in 30 successive phases, each phase amounting to 11 consecutive sessions, and each session including 40 choice trials. Three birds received a decreasing followed by an increasing sequence

of minimum VR components; the remaining three received an increasing followed by a decreasing series. For each session, the number of FR choices (defined as the number of entries in the FR terminal links) was recorded; we also recorded the rates of responding during the FR and VR terminal links.

RESULTS AND DISCUSSION

Session-by-session analysis. Figure 2 shows for each bird the total number of FR choices per session across the 330 sessions of the experiment. The left vertical axis indicates FR choices per session; that is, each data point represents the number of FR choices made during a single session. Because there were 40 choice trials per session, a data point at 40 indicates that the bird chose the fixed alternative on all trials (i.e., exclusive preference for the fixed schedule), and a data point at 0 indicates that the bird chose the variable alternative on all trials (i.e., exclusive preference for the variable schedule).

Although considerable variability can be seen within and among birds, the overall trends can be seen even in these fairly raw data. Preference for the FR schedule tended to track changes in the VR schedule composition. All of the birds in the increasing-decreasing group preferred the VR 60 schedule at the lower minimum component values, either immediately upon exposure or following a period of relative indifference. As the minimum component of the VR 60 increased and then decreased, a corresponding shift in preference for the fixed alternative occurred. Pigeon C3's magnitude of change in preference across phases was the least extensive, varying between 0 and 10 FR choices per session; however, even these shifts were systematically related to changes in size of the minimum VR component.

In the counterbalanced group (C7, C10, and C11), each bird's preference shifted systematically from the fixed to the variable alternative in the descending part of the series. In the subsequent ascending part of the series, C11 returned to preference for the FR, C10 showed a trend in that direction, and C7

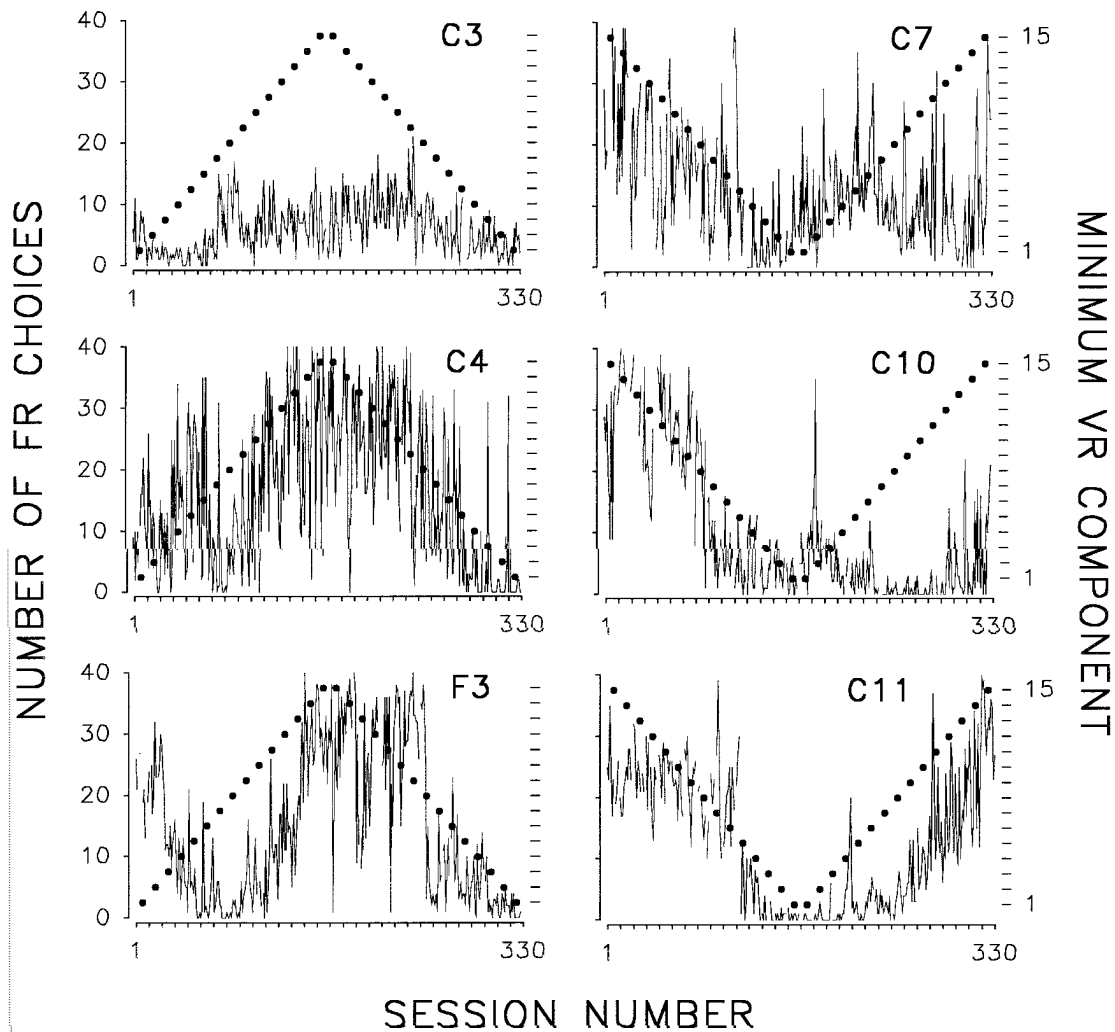


Fig. 2. Number of FR choices per session for each bird. Solid lines connect consecutive data points. (A few data points were lost during data storage.) The large solid dots represent the size of the minimum VR component in effect during each phase, as indicated by the right ordinate scale.

oscillated between the fixed and variable schedules.¹

To get a better grasp of variability at the session level, for each session, n , we computed the difference between the number of FR choices during this session and the number of FR choices during the immediately preceding session, $n - 1$. In principle, this differ-

¹The exposure of C7 and C10 to the final phase was continued with 11 additional sessions (not shown on graphs). C7's choices continued to oscillate, and C10 returned to almost exclusive preference for the FR, choosing the FR alternative at least 38 times per session in the final six of the additional 11 sessions.

ence can range from -40 to $+40$. We then computed autocorrelation coefficients (product moment) for this series of differences, from a lag of 1 to a lag of 10. For each bird (Figure 3, open squares), the autocorrelation coefficient was negative at a lag of 1, indicating that the birds' number of FR choices tended to oscillate from one session to the next; at higher lags, the autocorrelation coefficient moved toward 0. Furthermore, for each bird the absolute value of the autocorrelation coefficient was highest at a lag of 1 (Figure 3, solid lines) and moved toward 0 at higher lags. These absolute values may be

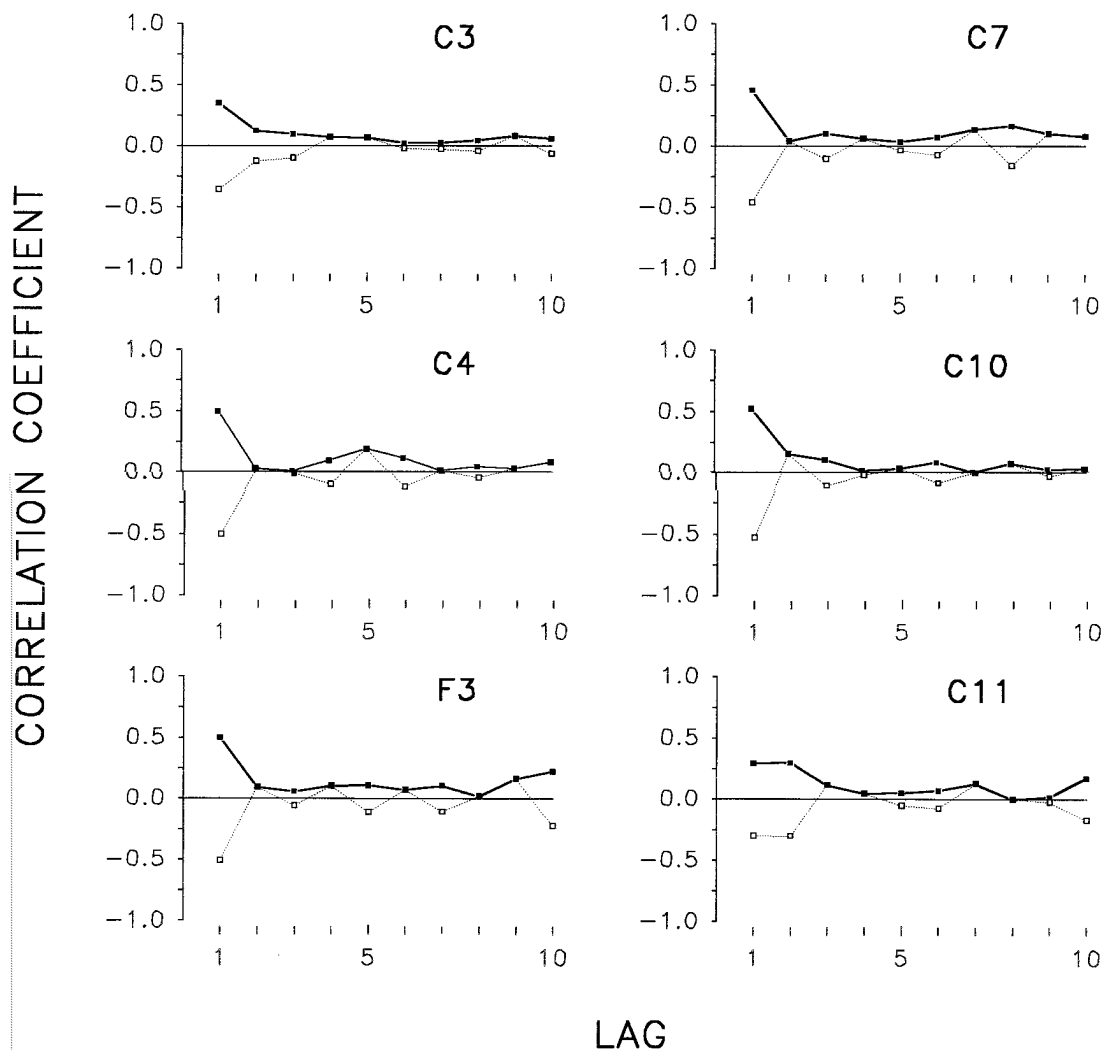


Fig. 3. Autocorrelation coefficients for the series of session-to-session differences in the number of FR choices (see text for more details). For each bird, the graphs present the signed values of the coefficients (dashed lines) as well as their absolute values (solid lines) as a function of the lag.

seen as measuring the strength of sequential dependencies across sessions, independently of their sign. Thus, Figure 3 shows that such dependencies became negligible beyond a span of roughly two (lag = 1) to three (lag = 2) sessions.

Within-phase analysis. Within-phase trends can be ascertained by comparing the average number of FR choices over the last five sessions of a given phase to the average number of FR choices over the first five sessions of the same phase. In principle, the difference between these two numbers, or within-phase dif-

ference (WPD for short), can range from -40 to $+40$. The WPDs showed many sign reversals from one phase to the next (Figure 4), producing no clear visual evidence that WPDs tended to be positive during an ascending series of VR distributions ($1 \rightarrow 15$) and negative during a descending series ($15 \rightarrow 1$). However, the mean ascending WPD (computed over the 15 phases of the ascending series) was positive for all birds except F3. (F3's negative value resulted from this bird's tendency to lag behind changes in the VR distribution, combined with a starting average of

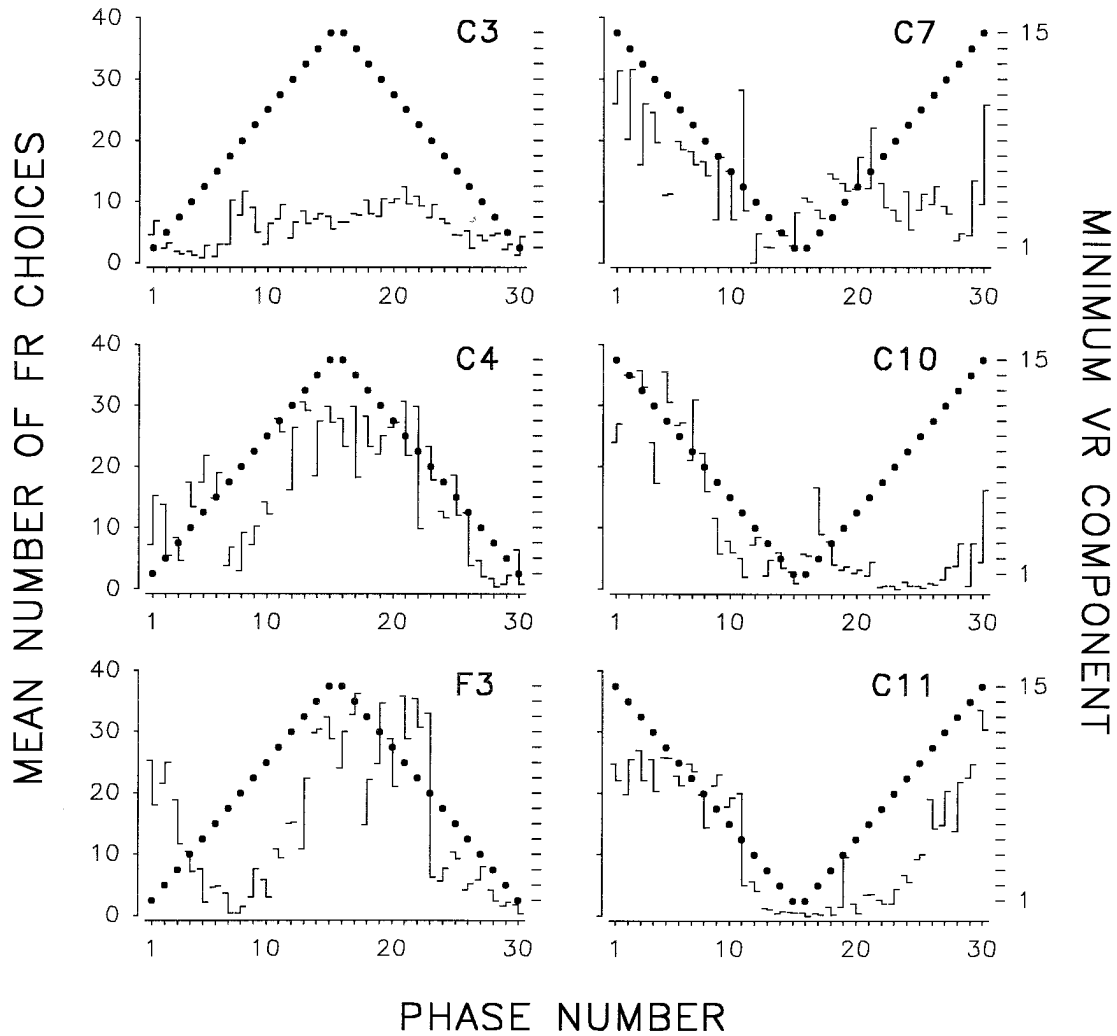


Fig. 4. Within-phase trends across phases for each bird. Each staircase motif represents a single phase; a vertical line connects the average number of FR choices over the phase's first five sessions (left branch) to the average number of FR choices over the phase's last five sessions (right branch). The large solid dots represent the size of the minimum VR component in effect during each phase, as indicated by the right ordinate scale.

Table 1
Mean within-phase differences for each subject.

Subject	Ascending	Descending	Difference
C3	1.19	-0.13	1.32
C4	1.58	-5.07	6.65
F3	-0.82	-1.19	0.37
C7	1.98	0.55	1.42
C10	0.94	-1.24	2.18
C11	0.93	-1.94	2.87

Note. The within-phase difference for a given phase is the difference between the average of the first five and the last five sessions, averaged separately for the ascending and the descending series.

FR choices around 20. F3's initial drop in the overall number of FR choices was accompanied by a high number of negative WPDs.) Conversely, the mean descending WPD (computed over the 15 sessions of a descending series) was negative for all birds except C7. Finally, the difference between the mean ascending WPD and the mean descending WPD was positive for all birds (Table 1). The small absolute values of these numbers (see Table 1) are consistent with the numerous sign reversals that were observed in the WPDs from one phase to the next.

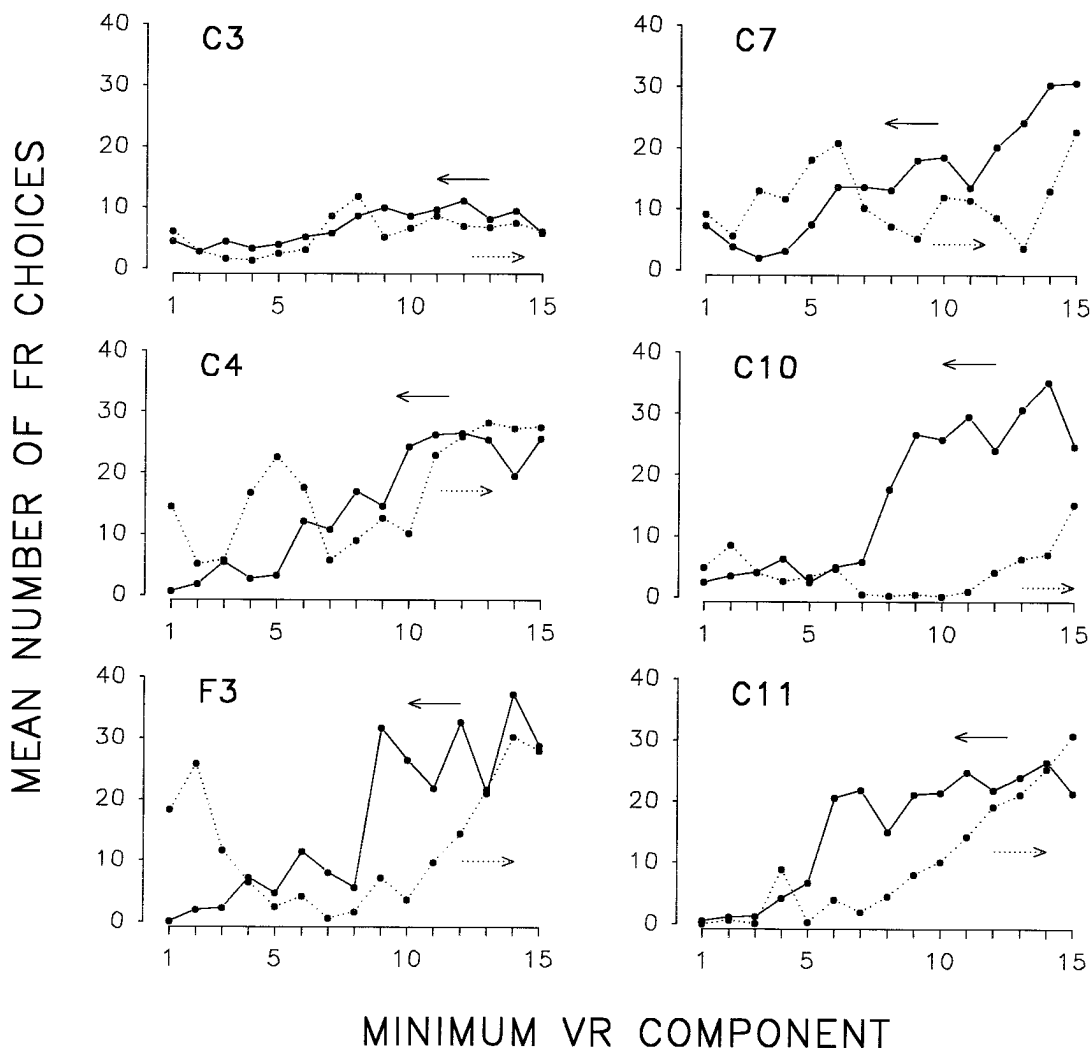


Fig. 5. Mean number of FR choices during the last six sessions of each phase, as a function of the minimum VR component in effect, shown separately for the ascending (dotted lines) and descending (solid lines) series for each bird.

Phase-by-phase analysis. In the remaining analyses, we focus on the average number of FR choices during the last six sessions of each phase. With plots of the average number of FR choices as a function of the minimum ratio component in effect, separately for the ascending (1 → 15) and descending (15 → 1) series, Figure 5 provides a picture of the transitions from low to high and from high to low numbers of FR choices; the most regular transitions are clearly S-shaped (e.g., C4 and C11, descending series). Figure 5 also provides evidence of historical effects across phases. For

each VR distribution, we computed the difference between the data point on the ascending curve (Series 1 → 15) and the corresponding data point on the descending curve (Series 15 → 1); these differences were then averaged across the 15 different VR distributions. The resulting hysteresis index is proportional to the between-curve area in Figure 5. For each bird except C4, the hysteresis index was positive (1.04, -2.39, 3.68, 3.14, 12.06, and 5.53 FR choices for C3, C4, F3, C7, C10, and C11, respectively). Stated otherwise, across phases the birds tended to

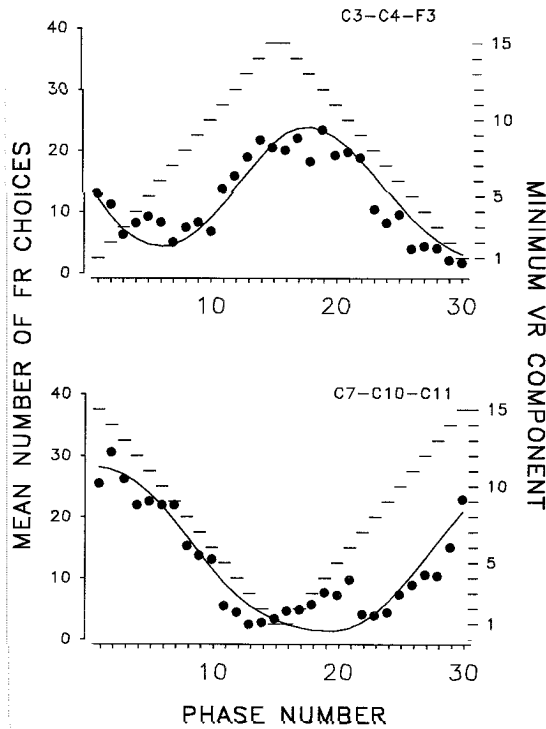


Fig. 6. Mean number of FR choices during the last six sessions of each phase (solid dots), averaged separately for each group of birds (Groups C3-C4-F3 and C7-C10-C11), and best fitting predictions (continuous lines; $\beta = .24$, $s = 7.06$). The short horizontal lines represent the size of the minimum component of the VR 60 distribution in effect during each phase, as indicated by the right ordinate scale.

persist in their previous behavior (C4's negative value arose merely from irregularities in this bird's ascending series). A bird starting a series with a low number of FR choices (Direction 1 \rightarrow 15) continued to emit relatively low numbers. Conversely, a bird starting with a high number of FR choices (Direction 15 \rightarrow 1) continued to emit relatively high numbers.

An interesting question is whether the hysteresis effect is itself asymmetrical. It is conceivable that the effects of exposure to a highly reinforcing VR schedule persist longer than the effects of exposure to a less reinforcing one. If this is indeed the case, then the upward limb of the ascending curve (1 \rightarrow 15) in Figure 6 should be displaced farther to the right than the downward limb of the descending curve (15 \rightarrow 1) is displaced to the left (e.g., see the data for C10). This hypoth-

esis was difficult to test, however, due to the fact that C4 and F3 began the experiment with relatively high numbers of FR choices (Figure 5, dotted lines). Assuming that behavioral momentum (Nevin, Mandell, & Atak, 1983) or inertia (Mazur, 1982) is actually indifferent to the path taken (1 \rightarrow 15 vs. 15 \rightarrow 1), a high starting point may nevertheless displace the whole ascending function toward the right, producing spurious evidence in favor of an asymmetrical process.

Model-based analysis. We modeled data at the phase level only. The measure of interest for each phase is a response proportion, the mean number of FR choices over the last six sessions divided by the total number of FR and VR choices per session (40 choices). To reduce variability, response proportions were averaged separately for the ascending-descending (C3, C4, and F3) and descending-ascending (C7, C10, and C11) groups. Averaging data from C3, C4, and F3 resulted in a smooth ascending-descending profile that was consistent with the overall shapes of the individual birds' transitions (despite a strong difference in preference range between C3 and the other birds). On the other hand, C7's oscillatory behavior during the ascending phase (Figure 1) proved to be more problematic, producing some heterogeneity in the averaged results of C7, C10, and C11 (discussed below).

At any rate, it is important to realize that the agenda for the following simulation was quite limited. In the absence of detailed dynamic information and hypotheses (cf. Marr, 1992), we did not aim at a complete account of the birds' behavior. Instead we used computer simulations to derive *qualitative* conclusions about the shape of the transitions and the relation between reinforcement and behavior in our experimental procedure (what made this possible was the relatively wide range of VR distributions used). In the simulations, we assumed that the response proportion during a phase depended jointly on the current schedule situation and on the subject's previous response proportions (cf. Figure 5). These assumptions were modeled with an exponentially weighted moving average (EWMA; Killeen, 1981). For any phase n except the first,

$$B_n = \beta S_n + (1 - \beta)B_{n-1}, \quad (2)$$

where β ($0 \leq \beta \leq 1$) is the currency parameter (denoting how the current schedule situation is weighted relative to previous situations) of the EWMA process (Killeen, 1981, 1994), B_n is the response proportion during phase n , B_{n-1} is the response proportion during the preceding phase, and S_n reflects the impact of the current schedule situation. When β is close to 1, behavior is almost exclusively influenced by the current situation; lower values of β imply a stronger contribution of previous behavior.

By definition in Equation 2, the overall behavioral effect S_n of the schedule situation reflects the reinforcing values of the FR and VR schedules. We assumed that these values, V_{FR} and V_{VR} , respectively, were adequately described by Mazur's (1984, 1987) general formulation (our Equation 1). But how do V_{FR} and V_{VR} combine to determine behavior? A first possibility is that S_n strictly matches the ratio of reinforcing values:

$$S_n = \frac{V_{FR}}{V_{FR} + V_{VR}}. \quad (3)$$

Strict maximizing, in contrast, implies that

$$S_n = \begin{cases} 1 & \text{if } V_{FR} > V_{VR}, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Neither strict matching of value nor strict maximizing could fit our data, however. The response proportions predicted by a strict matching hypothesis (Equation 3) stayed too close to indifference, whereas strict maximizing (Equation 4) tolerates only response proportions equal to 0 or 1. Clearly, most of our data fall somewhere between these two extremes. To locate the data on a matching-maximizing continuum (cf. Couvillon & Bitterman, 1985), we used a generalized matching approach (Baum, 1974). We assumed that

$$S_n = \frac{bV_{FR}^s}{bV_{FR}^s + V_{VR}^s}, \quad (5)$$

where b is a bias parameter and s represents the sensitivity of preference to relative reinforcement values (see Davison & McCarthy, 1988). Strict matching of value (Equation 3) implies no bias and a sensitivity equal to 1. An exponent s larger than 1 (overmatching) indicates deviations toward maximizing.

Aside from the preference level on the first

phase, Equations 1, 2, and 5 potentially include five free parameters: the value of food (A), the hyperbolic weighting parameter (k), the EWMA currency parameter (β), and the bias (b) and exponent (s) of the generalized matching equation. However, some of these parameters can be eliminated or estimated from the results of previous studies. First, the food value parameter, A , was deleted from Equation 1, because generalized matching implies that this parameter appears only in a ratio of the form $A^s/A^s = 1$ (see Equation 5). Second, the bias in Equation 5 was set equal to 1; in other words, we assumed that there was no bias in our data. Finally, k was set equal to 1.35, the best fitting value for Equation 1 in Mazur's (1986) study of ratio schedules. Aside from the starting level of preference, only two free parameters remain, the EWMA currency parameter (β) and the generalized matching exponent (s).

We fitted the mean data of Groups C3-C4-F3 and C7-C10-C11 with a single set of parameters (β , s). In order to apply Equation 1, FR and VR components were converted into delays, which implied dividing the response requirements by 2.80 responses per second (the average running rate during the terminal links, estimated from the birds' performances). Note that in Equation 1, $p_i = 1$ for the FR schedule and $p_i = .25$ for the VR schedule. In each group of birds, the initial value for B_n was set equal to the average of the response proportions observed on the first and second experimental phases. Best fitting parameters were then estimated by minimizing the root mean square deviation between observed and simulated numbers of FR choices (simulated response proportions times 40).

The best fit was obtained for $\beta = .24$ and $s = 7.06$ (Figure 6; percentage of variance accounted for = 84%). The main discrepancy between predictions and data arises from C7's strong oscillations in the ascending series, reflected in the averaged data of Group C7-C10-C11 (Phases 16 to 22, Figure 6, bottom panel). Removing C7's results from the data pool and fitting the model on Groups C3-C4-F3 and C10-C11 actually increased the percentage of variance accounted for to 90% (best fitting parameters: $\beta = .23$, $s = 9.04$). The model seemed to be consistent with the overall shape of the transitions, and exami-

nation of the residuals revealed no systematic trend.

Assuming, therefore, the rough validity of the EWMA approach and averaging simplifications, the high value of the exponent s ($\gg 1$) indicates approximate maximizing with respect to reinforcement value. Most of the time our birds completed their first link with one or two pecks; hence, the short FI 3 schedules of the choice phase were effectively functioning as FR 1 links (indeed, they were designed to function that way, while providing a bit of imposed exposure to the alternative stimuli). Our results are thus consistent with those obtained by Shull (1992) with a choice phase requiring a single peck and various FI schedules as terminal links. In Shull's study a generalized matching analysis relating choice behavior to relative reinforcement immediacies ($1/t_i$ in the notation of Equation 1) revealed approximate maximizing, with exponents ranging from 5.81 to 7.29 across birds (Shull, 1992, p. 41).

In contrast, Davison (1988, Experiment 1) and Davison and Smith (1986) studied concurrent chains with variable-interval schedules as the initial links and found biased matching to reinforcement value (Equation 5 with $s = 1$), value being defined as in Equation 1. Apparently, behavior on concurrent chains shows approximate matching or maximizing with respect to reinforcement value (defined as in Equation 1), depending on the nature of the initial links (interval vs. ratio schedules) or on their duration (cf. Davison, 1988, Experiment 2). Whether the overall pattern of results can be accounted for by a single theory, such as melioration theory (e.g., Vaughan, 1981, 1985), or some other quasi-dynamic model of the reinforcement process (Staddon, 1988), remains to be seen.

An especially interesting aspect of the data is the existence of historical effects that were operative at both session and phase levels (Figures 3 and 5, respectively). On the one hand, the observed session-to-session oscillations spanned successive days; on the other hand, the effects of a given VR distribution endured for weeks (or possibly even months). Our EWMA model addressed historical effects that were defined at the phase level, and with a currency parameter β equal to .24; but a EWMA process with β equal to .25 has also accommodated behavioral changes defined

over a few seconds (Killeen, 1994, Figure 5). The wide applicability of the EWMA formulation (with roughly similar β values) is consistent with the idea of behavioral processes organized across equally valid, multiple time scales (cf. Broadbent, Maksik, & Church, 1995²). Our choice of the EWMA model was based mainly on convenience and conceptual simplicity. Because the present results can also be conceptualized as a kind of persistence, the theory of behavioral momentum (Nevin et al., 1983) is a possible alternative to EWMA. Note, however, that both the behavioral momentum and the EWMA approaches summarize historical influences in a single quantity, an assumption that may eventually prove to be incorrect (see Miller & Matzel, 1987). The possibility remains, for instance, that different schedules (e.g., lean vs. rich VR schedules) affect behavior differently and with differing time courses. The hypothesis that the effects of a rich schedule persist longer than those of a leaner one could be tested by replicating our ascending ($1 \rightarrow 15$) and descending ($15 \rightarrow 1$) series, using experimentally naive birds and controlling more closely for the birds' starting levels of responding by adjusting the number of sessions in the initial condition until stability is reached.

Returning to a more purely descriptive analysis, the initial objective of the current project was to explore how parametric changes in the size of the smallest component within a variable schedule would contribute to shifts in preference for that schedule. The experiment provided clear evidence for the potency of these small values; given certain distributions, the variable schedule was consistently preferred over a fixed schedule with half its mean requirement. When exposed to these VR distributions, the pigeons behaved in a strongly nonoptimal fashion with respect to response expenditure (see Mazur & Vaughan, 1987), and continued to behave that way for several sessions within a phase, on the one hand, and even across successive phases, on the other. The dynamics of behavior across extended and multiple time scales

² Broadbent, H. A., Maksik, Y. A., & Church, R. M. (1995, May). *A fractal analysis of random interval data*. Paper presented at the meeting of the Society for Quantitative Analyses of Behavior, Washington, DC.

raise issues that go well beyond the original question.

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